On the wave speed of thermal radiation inside and near the boundary of an absorbing material

Bair V. Budaev and David B. Bogy
Department of Mechanical Engineering,
University of California, *Berkeley, CA 94720-1740, U.S.A.*
budaev@berkeley.edu, dbogy@berkeley.edu

Abstract

We argue that the conventional concept of a complex valued wave speed adequately describe propagation of signals, e.g. laser beams, but is not suitable for modeling thermal radiation in absorbing materials. Signal carrying waves pass some of their energy to the medium. This energy is eventually re-radiated, but in studies focused on the transmission of signals the re-radiated fields can be ignored. In order to study thermal radiation in an absorbing material, the material and the radiation must be considered together as a closed system whose energy is conserved, as well as its distribution between the material and radiation. The paper proposes a model of thermal radiation coupled with an absorbing medium to a closed, energy conserving system. This radiation field has normal modes, that correspond to an effective speed. Assuming an absorbing material and the radiation in it are in thermal equilibrium we show that deep inside the material the average speed of photons is given by a frequency and temperature dependent expression, which at high frequencies approaches the speed of light in vacuum while at low frequencies it approaches the half of this value. We further show that closer to the boundary of the medium the speed of thermal radiation depends in a complex way on the refractive index and the extinction coefficient of the material, as well as the direction of propagation and the distance from the surface. Finally, we discuss how the issues described here relate to radiation heat transfer in nanoscale systems.

1 Introduction

Charged subatomic particles in thermal motion generate electromagnetic fields that provide the means for different forms of heat transport. The short-range quasi-static components of the electromagnetic fields are responsible for the mechanism of heat transport known as heat conduction, and the long-range fields generated by accelerating charges are responsible for radiative heat transport [1]. Thermal radiation is singled out from other mechanisms of heat transport by its ability to carry noticeable heat across vacuum gaps wider than a fraction of a micron.

Since the late 1800s studies of radiative heat transport have used the concept of a "black body", which despite being an abstraction, often provides a reasonable approximation for real materials. A black body is an idealistic material object that exchanges energy with the exterior world only through electromagnetic radiation, absorbs all external radiation that reaches the body, and renders the outgoing radiation completely uncorrelated with the absorbed incoming radiation. The importance of the black body concept is related to the universal character of its radiation spectrum, which is determined solely by its temperature.

Assume that a black body at temperature T has a vacuum cavity with a large volume V. Then, the spectrum of thermal radiation into the cavity is described by Planck's formula

$$W(\omega,T) = 2D_0(\omega)\widetilde{M}^*(\omega,T)\hbar\omega, \tag{1}$$

where $W(\omega,T)\mathrm{d}\omega$ is the energy of the electromagnetic field in the cavity per unit volume with frequencies from the band $(\omega,\omega+\mathrm{d}\omega)$; the factor "2" takes into account the presence of electromagnetic fields of two different polarizations;

$$D_0(\omega) = \frac{\omega^2}{2\pi^2 c_0^3}, \tag{2}$$

is the density of states defined such that $D_0(\omega)\mathrm{d}\omega$ represents the number of normal modes of the electromagnetic field of a single polarization in a unit volume of a large vacuum cavity; c_0 is the speed of light in vacuum; and

$$\widetilde{M}^*(\omega,T) = \frac{1}{\mathrm{e}^{\hbar\omega/\kappa T}-1}, \tag{3}$$

is the average energy level of a single normal mode at frequency ω considered as a quantum harmonic oscillator in equilibrium with a medium at temperature T, and $\hbar\omega$ is the energy of one photon at frequency ω. The energy of a quantum harmonic oscillator at frequency ω is known to take values from the arithmetic series $E_M(\omega) = E_0(\omega) + M\hbar\omega$ with the increment $\hbar\omega$ and a non-negative index $M \geq 0$. This index is referred to as the energy level of the oscillator, and it is customary to say that an oscillator on the M-th level contains M indistinguishable oscillators, each carrying the energy $\hbar\omega$. The energy levels of normal modes at frequency ω of the radiative field in equilibrium at temperature T randomly fluctuate around the average (3) provided by Planck's theory, [2].

The Planck law (1) is universal in the sense that it is valid for any large vacuum cavity in any "black body". However, this formula provides no information about the radiation from a black body into a cavity filled by a medium, as well as about the radiation field inside the black body itself, which can be treated as a field in a cavity filled by the material of the black body. In order to see why Planck's law (1) cannot be straightforwardly applied to radiation into an arbitrary medium it suffices to analyze a conventional derivation of this law [3, 4].

Any electromagnetic field in a cavity admits a decomposition into normal modes, each of which can be treated as a harmonic oscillator at a certain frequency ω. If the normal modes are in thermal equilibrium at temperature T, then the average energy of each mode at frequency ω has the value $\mathcal{E}(\omega,T) = \widetilde{M}^*(\omega,T)\hbar\omega$, with $\widetilde{M}^*(\omega,T)$ from (3) representing the average number of indistinguishable photons carried by a single mode. The number of normal modes is provided by the Weyl asymptote

[5, 6, 7] which states that as the volume of the cavity increases, the number density $\mathrm{d}N(k)$ of normal modes of the reduced wave equation $\nabla^2\phi = -k^2\phi$, with k in the interval $(k, k + \mathrm{d}k)$ is estimated as

$$\mathrm{d}N \approx \frac{k^2 \mathrm{d}k}{2\pi^2}. \tag{4}$$

Since in vacuum the wavenumber k and the frequency ω are related by $k = \omega/c_0$, the number density of normal modes with frequencies from the range $(\omega, \omega + \mathrm{d}\omega)$ is estimated as

$$\mathrm{d}N(\omega) \approx D_0(\omega)\mathrm{d}\omega. \tag{5}$$

Finally, multiplying the average energy density of a single normal mode $\widetilde{M}^*(\omega, T)\hbar\omega$ by twice the number density $\mathrm{d}N(\omega)$ of normal modes of a single polarization we get the Planck law (1).

The transition from (4) to (5) relies on the relation $k = \omega/c_0$ between the wave vector k and the frequency ω of electromagnetic waves in vacuum. In the case of radiation into a medium with a frequency-dependent speed $c(\omega)$, the wave number is defined as $k = \omega/c(\omega)$, and (4) reduces to

$$\mathrm{d}N(\omega) = \frac{\omega^2}{2\pi^2 c^2(\omega)} \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{\omega}{c(\omega)}\right) \mathrm{d}\omega = \frac{\omega^2\, \mathrm{d}\omega}{2\pi^2 c^2(\omega) v(\omega)} \tag{6}$$

where

$$v(\omega) = 1 \Bigg/ \frac{\mathrm{d}(\omega/c(\omega))}{\mathrm{d}\omega}\ , \tag{7}$$

is the group speed of light in the medium [8]. Therefore, the power spectrum of black body radiation into a dispersive medium with the phase speed of light $c(\omega)$ is represented by the formula

$$W(\omega, T; c(\omega)) = 2D(\omega, c(\omega))\mathcal{E}(\omega, T), \tag{8}$$

which is similar to (1), but with a more complex density of states

$$D(\omega, c(\omega)) = \frac{\omega^2}{2\pi^2 c^2(\omega) v(\omega)}. \tag{9}$$

Since the assumption $c(\omega) = c_0$ reduces (7) to $v(\omega) = c_0$ and then (8) to (1), we see that (8) appears as an extension of Planck's law describing black body radiation into a dispersive medium.

The extended Planck's law (8) is derived for any differentiable phase speed $c(\omega)$ of light in a medium accepting the radiation. Nevertheless, straightforward applications of (8) to practical cases are not possible. Indeed, the conventional formula for the speed of light in a medium

$$c(\omega) = \frac{c_0}{n(\omega)}, \tag{10}$$

includes the refractive index $n(\omega)$, which is either a constant or has an imaginary part [8]. If $n(\omega)$ is a positive constant then (8) reduces to (1) with c_0 replaced by $c = c_0/n$. However, if $\mathrm{Im}[n(\omega)] \neq 0$, then formulas (6) and (8) are meaningless because neither the number of the normal modes nor the energy density can take complex values.

Since real materials have frequency-dependent refractive indices, the last observation indicates that (8) may have no potential for applications. However, a comparison of the derivation of (8) with the origin of (10) discussed in many textbooks, e. g. in [3, 8], reveals that the speeds of light in the medium in (8) and (10) have different physical meanings, which implies that in order to use (8) it is necessary to find an appropriate definition of the speed of light $c(\omega)$ in a medium. Thus, as discussed in the next section, $c(\omega)$ from (10) characterizes the speed of propagation of electromagnetic signals through a medium, but $c(\omega)$ that appears in (6) – (9) is used solely to compute the number density of normal modes of the domain's oscillations.

2 Electromagnetic signals *vs* thermal radiation

Let an electromagnetic wave excited by some source, such as a laser or a radio transmitter, propagate through a medium. This wave interacts with atoms of the medium causing changes both in the medium and in the wave itself. The classical theory of electromagnetic wave propagation through a medium is based on an assumption that any material consists of positively charged nuclei and negatively charged electrons that stay at rest in the absence of an external electromagnetic field, start moving when such field is applied, and while moving radiate electromagnetic waves whose interference with the initial field forms a total field in the material, [3, 8].

Since an electron is much lighter than any positively charged particle it is customary to ignore the motion of the nuclei and focus on the motion of electrons considered as particles with the mass m_e and the charge $-q_e$, whose interactions with the electromagnetic field are governed by the Lorentz theory of electrons, [9], based on a classical concept of a damped mechanical oscillator.

Let an electron be moved from rest at time $t = t_0$ by an electric field

$$\vec{E}(\omega, t) = A_0 \eta(t - t_0) \cos(\omega t) \vec{e}, \qquad \eta(t) = \begin{cases} 0, & t < 0; \\ 1, & t \geq 0, \end{cases} \tag{11}$$

with a constant amplitude A_0 and with a polarization along a unit vector $\vec{e}$. Then, the displacement $\xi(t)$ of the electron along $\vec{E}$ can be described by a Newtonian equation

$$m_e \frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} = -A_0 q_e \eta(t - t_0) \cos(\omega t) - m_e \omega_0^2 \xi - m_e \Gamma \frac{\mathrm{d}\xi}{\mathrm{d}t}, \tag{12}$$

where the first term on the right-hand side represents the force exerted on the electron by the electric field, the second term represents the "spring" force proportional to a small displacement of the electron from its resting position, and the last term represents frictional forces related with damping of the electron's energy to interactions with nuclei and other electrons, which are not directly involved in (12). The motion controlled by (12) converges to forced oscillations, [10],

$$\xi(t) = \frac{-A_0 q_e \cos(\omega t + \beta)}{m_e\sqrt{(\omega^2 - \omega_0^2)^2 + \Gamma^2\omega^2}}, \qquad \beta = \arctan\left(\frac{\Gamma\omega}{\omega^2 - \omega_0^2}\right), \tag{13}$$

which draw energy from the supplied field and convert it to heat at the average rate

$$I(\omega; \omega_0, \Gamma) = \frac{A_0^2 q_e^2 \Gamma \omega^2}{2m_e\left[(\omega^2 - \omega_0^2)^2 + \Gamma^2\omega^2\right]}. \tag{14}$$

Since the energy of oscillations (13) remains constant, the last expression also describes the rate at which the supplied field looses its energy. The energy used to overcome the frictional forces exerted on the electron by other particles is eventually re-radiated by these particles, but, since the locations, speeds, masses and other parameters of the particles that caused the frictional forces may be considered as random and independent of the applied field, the re-radiated field appears as random noise, which cannot be identified as a part of the supplied field (11).

Let an electromagnetic wave $\vec{E}_0(\omega, t, x) = A_0\eta(t)\cos(\omega[x/c_0 - t])\vec{e}_0$ arrive from vacuum and enter an absorbing material at the point $x = x_0$. As it moves through the material, the number of electrons engaged in the motion controlled by (12) increases proportionally to the travelled distance $L = |x - x_0|$. Since every electron dissipates energy at the rate (14), which is proportional to the energy of the wave at the electron's locations, the energy of the propagating wave decays proportionally to $\mathrm{e}^{-p_0 L}$, where $p_0 > 0$ and L is the traveled distance. Since the energy density of a wave is proportional to the square of its amplitude, the amplitude A_L of the wave after passing a distance L inside the material decays as $A_L = A_0\mathrm{e}^{-\sigma L}$, where $\sigma = \frac{1}{2}p_0 > 0$ characterizes the rate of absorption. Electrons performing forced oscillations (13) radiate electromagnetic waves, but, unlike the noise, these waves are correlated with the initial wave and interfere with it, forming a total field whose structure $A_0\mathrm{e}^{-\sigma L}\cos\left(\omega[L/c - t]\right)\vec{e}$ mimics the structure of the incident wave, but has an exponentially decaying amplitude and a different phase speed c, [3]. According to the outlined theory, a recognizable incident wave $\vec{E}_0(\omega, t, x)$, which enters a material body at $x = x_0$, is transformed to another recognizable wave $\vec{E}(\omega, t, x) = \mathrm{e}^{-\sigma|x - x_0|}\cos\left(\omega(|x - x_0|/c(\omega) - t]\right)\vec{e}$ propagating inside the body with a different phase speed $c(\omega)$ and with a decaying amplitude $A_0\mathrm{e}^{-\sigma|x - x_0|}$. The waves propagating in the vacuum and in the material are strictly correlated and can be considered as continuations of each other.

The propagation of signals carried by electromagnetic waves admits an alternative illustration in terms of photons. In this model a spatially localized wave packet with the dominant frequency ω is treated as a cloud of photons each carrying the energy $\hbar\omega$. For simplicity we limit ourselves to a one-dimensional case and assume that the packet at time $t = 0$ is localized around the point x and contains m photons, so that the entire packet carries energy $m\hbar\omega$.

In vacuum the photons travel uninterrupted with the speed c_0, and the packet moves as a rigid body, whose shape can be described by the function $m(x/c_0 - t)$, as shown in the left part on Fig. 1. When this packet enters a medium, individual photons can be absorbed and scattered. Let p_0 be the probability that a photon disappears from the packet before it travels a unit distance. Then the number of photons in a packet traveling the distance x appears as a random variable distributed by the Poisson law with the coefficient p_0, which implies that on average the number density of photons decays proportionally to $\mathrm{e}^{-p_0 x}$. Let τ be the average time during which a multiply scattered photon travels a unit distance along the direction of the initial wave. Then, $c = 1/\tau < c_0$ represents the average speed of the packet's propagation through the medium. Taking into account that the number of photons in the packet decays proportionally to $\mathrm{e}^{-p_0 x}$, we conclude that the shape of a packet is described by the function $\mathrm{e}^{-p_0 x} m(x/c - t)$, where c is the average speed of the packet and p_0 is the rate of absorption of photons traveling a unit distance.

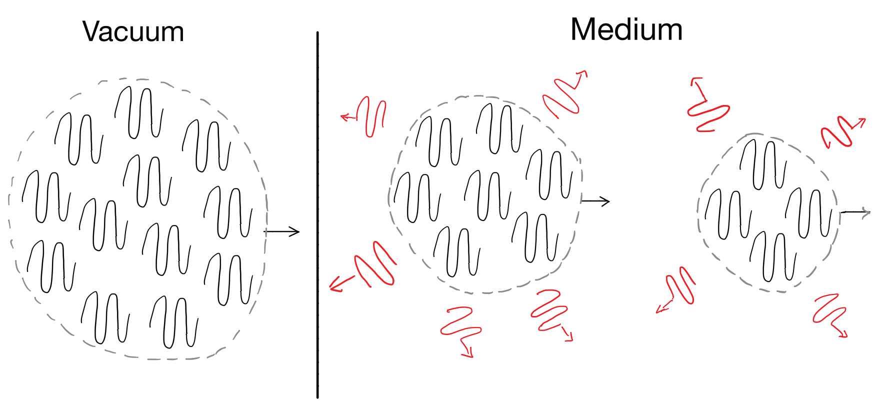


Figure 1: Disappearance of photons from the packet traveling through a medium

The above shows that the models of propagation of electromagnetic waves through a medium in terms of plane waves and in terms of quasi particles agree in that the energy of waves decays exponentially with the travelled distance and that the speed of propagation in a medium is different than in vacuum. Such behavior admits a convenient description in terms of the exponential representations of plane waves. Consider a monochromatic plane wave propagating in vacuum along the x-axis and described by the amplitude $u_0(x, t; \omega) = \mathrm{Re}\left(\mathrm{e}^{\mathrm{i}\omega(x/c_0 - t)}\right)$. When this wave enters a medium its exponentially decaying amplitude can be described by $u(x, t; \omega) = \mathrm{e}^{-\sigma x}\,\mathrm{Re}\left(\mathrm{e}^{\mathrm{i}\omega(x/c - t)}\right)$, which is equivalent to

$$u(x, t; \omega) = \mathrm{e}^{-\sigma x} \cos\left(\frac{\omega x}{c(\omega)} - \omega t\right) \equiv \mathrm{Re}\left(\mathrm{e}^{\mathrm{i}\omega(n(\omega)x/c_0 - t)}\right), \tag{15}$$

where $n = n' + \mathrm{i}n''$ is a complex number related with σ and c by the formulas

$$c = \frac{c_0}{n'}\,, \qquad \sigma = \frac{\omega n''}{c_0}. \tag{16}$$

The real and the imaginary components of $n = n' + \mathrm{i}n''$ are usually referred to as the refraction index and the extinction coefficient of the material, respectively.

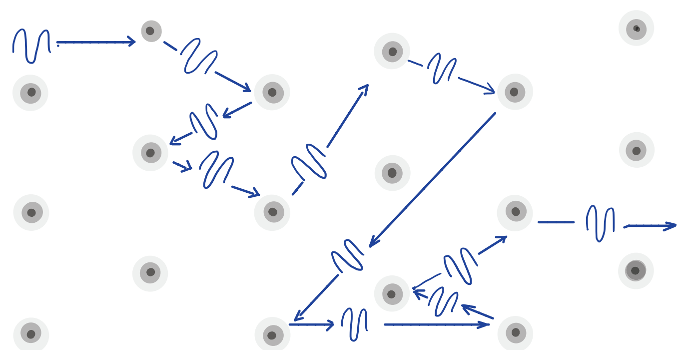


Figure 2: Reduction of photon's speed in a medium

In both considered models, electromagnetic waves change the state of the material, but in studies focused on the propagation of waves, such as radio signals, the changes in the matter may often be ignored, and (16) adequately describes the evolution of a signal in a medium with a complex refractive index $n = n' + \mathrm{i}n''$. However, these interpretations of wave propagation and absorption are not suitable for thermal radiation, which does not carry distinguishable signals.

In order to elaborate on the last statement we observe that subatomic particles, e.g. electrons and nuclei, are never still, as assumed in the outlined Lorentz model of the response of matter to an external electromagnetic field. On the contrary, subatomic charges permanently move and sporadically radiate photons even if no external electromagnetic field is applied. These photons have random frequencies, move in random directions and at random times they are absorbed by the matter, which gets back the energy of the previously emitted photons. In thermal equilibrium the rates of photon absorption and emission are equal, so that the average energy densities of the electromagnetic field and of the medium remain constant. In this situation it is inappropriate to talk about attenuation of thermal radiation because the average energy of the total field remains constant, similarly to how the average energy of all particles of a gas remains constant while the energies of individual particles change after each collision.

Obviously, the average speed of photons in a model where photons are not grouped into distinguishable packets, cannot be defined by the formula (10) with a complex-valued refractive index, which describes the evolution of groups of photons traveling together. It is still possible to treat any photon as a sole representative of a group, and, therefore, to describe its evolution in terms of the conventional model of interaction of an electromagnetic field with the matter. However, since thermal radiation includes photons with arbitrary parameters, the number of individual photons makes it impossible to consider each of them separately, which effectively makes it impossible to use the conventional approach and forces us to rely on a statistical description of the ensemble of photons, considered to be independent of each other, which eliminates chances to consider groups of photons traveling together. This suggests that in order to analyze thermal radiation it is appropriate to model the thermally excited electromagnetic field as a gas of photons with time independent distributions of their energy, direction and the speed, and to study these distributions.

3 The average speed of thermally excited photons

Photons at frequency ω can be represented by plane-wave solutions of Maxwell's equations that are normalized to have the average energy density $\hbar\omega$, [11]. In particular, such a photon can be represented by a wave described by its scalar and vector potentials

$$\phi = 0, \qquad \boldsymbol{A}_{[1]}(\boldsymbol{x}, t; \omega, \boldsymbol{e}, \boldsymbol{d}, \alpha) = \boldsymbol{d}\sqrt{\frac{\hbar}{\epsilon_0 \omega}} \cos\left(\frac{\omega}{c}\boldsymbol{e}\cdot\boldsymbol{x} - \omega[t-\alpha]\right), \tag{17}$$

where ϵ_0 is the permittivity of vacuum, α is a time shift, $\sqrt{\hbar/\epsilon_0\omega}$ is a normalization factor, $\boldsymbol{e}$ is a unit vector along the direction of propagation, and $\boldsymbol{d}$ is a unit vector normal to $\boldsymbol{e}$, which determines the polarization. As for the subscript [1] it emphasizes that this formula represents a single photon.

Photons with different frequencies, polarizations, directions, and time shifts are distinguishable from each other. However, photons with identical sets of these parameters appear as indistinguishable particles whose collective behavior is governed by the Bose-Einstein statistics, [12, 13]. In order to emphasize that indistinguishable photons must be considered together, we reserve the symbol $\boldsymbol{A}_{[M]}(\boldsymbol{x}, t; \omega, \boldsymbol{e}, \boldsymbol{d}, \alpha)$ for a set of M indistinguishable photons with the parameters ω, $\boldsymbol{e}$, $\boldsymbol{d}$, and α. Then, a gas of photons can be viewed as a superposition

$$\sum_{j,k,l,\nu} \boldsymbol{A}_{[M]}(\boldsymbol{x}, t; \omega_j, \boldsymbol{e}_k, \boldsymbol{d}_l, \alpha_\nu), \qquad M = M(\omega_j, \boldsymbol{e}_k, \boldsymbol{d}_l, \alpha_\nu), \tag{18}$$

that includes groups of $M(\omega_j, \boldsymbol{e}_k, \boldsymbol{d}_l, \alpha_\nu)$ indistinguishable photons with frequencies ω_j, directions $\boldsymbol{e}_k$, polarizations $\boldsymbol{d}_l$ and phase shifts α_ν.

In vacuum photons propagate freely and their superposition (18) remains unchanged over time. In a medium these waves interact with atoms, which may absorb or emit photons. The absorption of one photon corresponds to the replacement in (18) of the corresponding index M by $M-1$. Conversely, the emission of a photon corresponds to the replacement in (18) of the index M by $M+1$, and it is well known that a system with M indistinguishable photons has $(M+1)/M$ times more chances to get a new photon than to lose one, [11]. This property makes it possible to estimate the average speeds of photons in an absorbing material maintained at a fixed temperature.

Assume that the photon's absorptions and emissions occur at random times distributed by the Poisson's law with the average rates of occurrences $P_{\rm abs}$ and $P_{\rm emit}$, respectively, [14, 15]. Then, the properties of the Poisson processes imply that the inverses $1/P_{\rm abs}$ and $1/P_{\rm emit}$ represent the average times before the photon's nearest absorption and re-emission. On the other hand, a photon's absorptions and emissions can be considered from a different prospective as the interruptions and the beginnings of its motion. Adopting this point of view we see that $1/P_{\rm abs} = \tau_{\rm move}$ and $1/P_{\rm emit} = \tau_{\rm stop}$ can be interpreted as the average times of the photon's motion until it stops and of the photon's stoppage time until it starts moving again. Then, assuming that a photon moves with the speed c_0 between its re-emissions and absorptions, we represent the average speed of a photon as

$$c_* = \frac{c_0 \tau_{\rm move}}{\tau_{\rm move} + \tau_{\rm stop}} = \frac{c_0 P_{\rm emit}}{P_{\rm emit} + P_{\rm abs}} = \frac{c_0}{1 + P_{\rm abs}/P_{\rm emit}}. \tag{19}$$

The above mentioned property of photon statistics leads to the estimate

$$\frac{P_{\rm abs}}{P_{\rm emit}} = \frac{\widetilde{M}^*}{\widetilde{M}^* + 1}, \tag{20}$$

where $\widetilde{M}^*$ is the average number of indistinguishable photons with a fixed set of parameters $(\omega, \boldsymbol{e}, \boldsymbol{d}, \alpha)$. Assuming that the material and radiation are in thermal equilibrium at temperature T we represent $\widetilde{M}^*$ by (3) and reduce (19) to the expression

$$c_*(\omega, T) = \frac{c_0}{1 + \mathrm{e}^{-\hbar\omega/\kappa T}}, \tag{21}$$

describing the average speed of photons at frequency ω in the black body at temperature T.

Obviously, the speed (21) is not constant. At the low frequency limit $\omega \to 0$ it approaches the temperature-independent value $c_*(0, T) = \frac{1}{2} c_0$, which means that low-frequency photons spend equal times propagating and being absorbed by the matter. As the frequency increases, $c_*(\omega, T)$ monotonically approaches another temperature-independent limit $c_*(\infty, T) = c_0$, which conforms with the expectation that extremely short waves go through the material without interruptions.

It is often convenient to characterize electromagnetic waves by their wavelength λ_0 in vacuum instead of the frequency ω. Taking into account that $\omega\lambda_0 = 2\pi c_0$ we convert (21) to the form

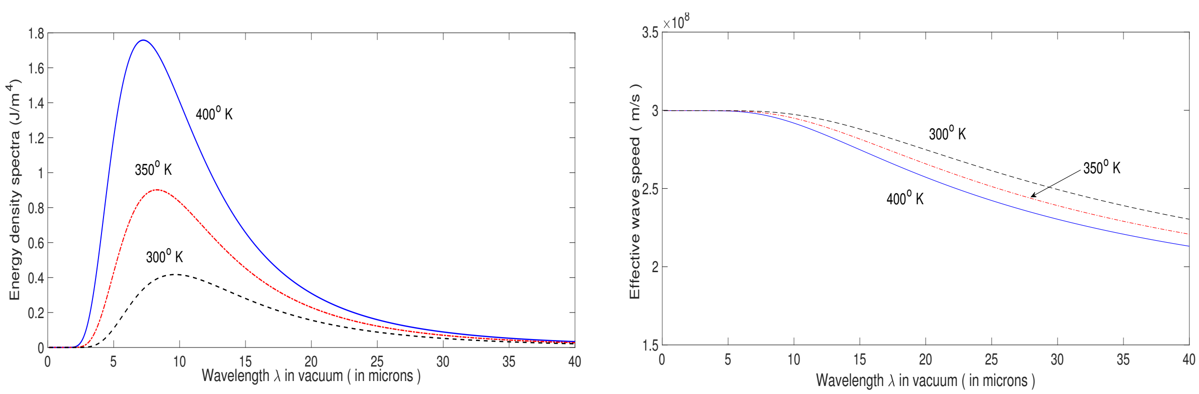
$$c_*(\lambda_0, T) = \frac{c_0}{1 + \mathrm{e}^{-2\pi c_0 \hbar/\lambda_0 \kappa T}}. \tag{22}$$

Then, the effective wavelength λ_* of radiation inside a black body is estimated as

$$\lambda_* \equiv \lambda_*(\lambda_0, T) = \frac{\lambda_0}{1 + \mathrm{e}^{-2\pi c_0 \hbar/\lambda_0 \kappa T}}. \tag{23}$$

Although (22) and (23) are merely equivalents of (21), the representations of the wave speed and wavelength of thermally excited waves inside materials in terms of the wavelength in vacuum are convenient for the characterization of the domain of applicability of the obtained formulas.

To get an idea about the potential implications of (22) and (23) we combine in Fig. 3 graphs of power spectra of thermal radiation with graphs of the effective speed of thermal radiation. The subfigure a) shows Planck's spectra at temperatures 300K, 350K and 400K of black body radiation into vacuum, plotted against the wavelength in vacuum; and the subfigure b) shows the effective speed $c_*(\lambda, T)$ of thermal photons inside black bodies at temperatures 300K, 350K and 400K, which are also plotted against the wavelength in vacuum. These subgraphs show that although the effective speed of radiation inside a black body depends on the wavelength, in the dominant part of the spectra, i. e. approximately from 3 to 20 microns, the reduction of the effective speed of radiation is limited to about 16% to 23% of the maximal possible speed.



a) Energy density spectra (J/m^4, *per unit volume, per unit wavelength) of thermally excited photons as functions of the wavelength* (10^{-6}m) *at temperatures 300K, 350K, 400K*

b) Effective speeds (m/s) *of thermally excited photons as functions of the wavelength* (10^{-6}m) *at temperatures 300K, 350K, 400K.*

Figure 3: Speed and energy spectra of thermally radiation in a black body

4 Thermal radiation near a boundary of a medium in thermal equilibrium

In the above we concluded that a thermally excited electromagnetic field inside an absorbing medium can be modeled by an ensemble of photons propagating with the speed c_* from (21) or (22), which appears to be independent of the material. This conclusion is based on the assumption that each photon is absorbed and re-emitted infinitely many times, which is certainly the case deep inside an absorbing material, but which is not valid near its boundary. It is in this boundary layer that the speed becomes dependent on the material.

Consider photons passing a point with Cartesian coordinates $(x, 0, 0)$ located inside the half-space $x > 0$ occupied by an absorbing material, as shown in Fig. 4. These photons can be subdivided into two distinctive groups. The first group includes photons that reach the surface or arrive from it in one excursion, without having been absorbed and re-emitted, but with possible scatterings that merely change the photon's speed. The second group includes photons that reach the boundary or come from it after one or more absorptions and re-emissions. For simplicity we assume that photons from the second group propagate with the speed c_* from (21) or (22), derived for photons experiencing many absorption and re-emissions, which eliminate chances to trace the trajectories of individual photons. As for photons from the first group, they propagate without absorptions and re-emissions, which means that they can be traced individually as signal carrying waves with the speed $c(\omega) = c_0/n'(\omega)$ determined by the refractive index of the material $n'(\omega)$. Since such photons reach the boundary without having been absorbed, we assume, in agreement with the photon's model of signal carrying waves, that the amplitudes of waves represented by these photons do not decay. Then, at the depth x inside the body the average speed of photons at frequency ω propagating at the angle θ with respect to the x-axis can be represented as

$$v(x, \omega, \theta) = \gamma(\omega, L)c(\omega) + (1 - \gamma(\omega, L))c_*(\omega), \qquad L = \frac{x}{\cos\theta}, \tag{24}$$

where $\gamma(\omega, L)$ is the probability that a photon at frequency ω travels the distance L without having been absorbed.

In order to estimate the probability $\gamma(\omega, L)$ in (24) we assume that photon's absorptions as random events occur with the probability $p_0\Delta L$ while a photon travels a small distance ΔL. Such random processes are referred to as Poisson processes, [14, 15], and it is known that the probability of traveling a distance L without absorption has the value $e^{-p_0 L}$. On the other hand, as discussed in Section 2, when a packet of photons travels through a material with the extinction coefficient $n''(\omega)$,

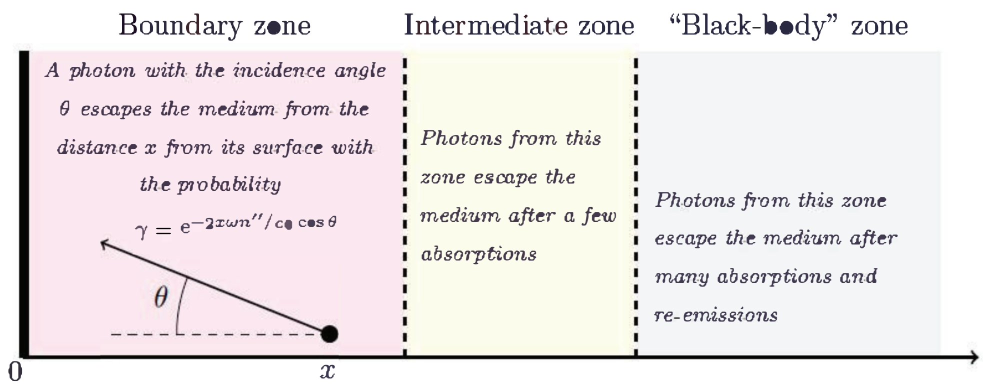


Figure 4: Photons near the boundary

the number of photons in a packet decays proportionally to $e^{-2L\omega n''/c_0}$. Therefore, the comparison of the two observations suggests that the probability $\gamma(\omega, L)$ in (24) has the value

$$\gamma(\omega, L) = e^{-2\sigma(\omega)L}, \qquad \sigma(\omega) = \frac{\omega n''}{c_0}. \tag{25}$$

Finally, combining (24) and (25) we find that the average speed $v(\omega; x, \theta)$ of photons at the depth x under the surface of a material half-space can be estimated by

$$v(x, \omega, \theta) = c_0 \left(\frac{e^{-2\omega n''(\omega)x/c_0 \cos\theta}}{n'(\omega)} + \frac{1 - e^{-2\omega n''(\omega)x/c_0 \cos\theta}}{1 + e^{-\hbar\omega/\kappa T}} \right), \tag{26}$$

where θ is the incidence angle of the photons, while $n'(\omega)$ and $n''(\omega)$ are the real and imaginary components of the complex refraction index $n(\omega) = n'(\omega) + in''(\omega)$ of the material.

The speed (26) of thermal radiation near the boundary of the medium depends in a rather complex way on the refractive index and the extinction coefficient of the material, as well as on the direction of propagation and the distance from the material's surface. This speed is pushed towards the universal, material-independent value (21) by any of the following changes of the parameters: the extinction coefficient $n''(\omega)$ increases; the distance x from the surface increases; the wavelength of radiation $\lambda = 2\pi c_0/\omega$ decreases; and the incidence angle θ increases. In the case when the extinction coefficient vanishes, $n'' = 0$, the complex refractive index $n = n' + in''$ takes the real value $n \equiv n'$ and (26) reduces to $v = c_0/n$ provided by the formula (10), which can be derived by the classical method outlined in Section 1 and is limited to non-absorbing materials.

In order to illustrate the obtained results we consider silicon dioxide and gold, which are both widely used in electronic devices but have very different properties. The refractive indices $n'(\lambda_0)$ and extinction coefficients $n''(\lambda_0)$ of these materials are plotted in Fig. 5 and Fig. 6 by dashed and solid lines against the wavelength of light in vacuum λ_0. Fig. 5 shows that the absorption of silicon dioxide is very different in three distinctive parts of the spectrum: this material is almost transparent for radiation with wavelengths between 0.14 μm and about 7 μm; is noticeably absorbing in the wavelength bands around 9 μm and 21 μm; and is moderately transparent in other regions. This suggests that in the band characterized by high absorption the effective wave speed of radiation may deviate from the universal value only in a very thin boundary layer, and that in the other bands, the transitional boundary layer may be considerably wider. On the contrary, the dependence of the absorption of gold on the wavelength is rather straightforward: in the band $\lambda < 500$ nm it increases almost monotonically, and in the band $\lambda > 500$ nm the increase becomes nearly linear. However, the refractive index of gold has a noticeable dive below unity in the optical band between 0.5 μm and 1μm, so that the phase speed of light in this band may exceed the speed of light in vacuum. Such phenomenon is not unusual and often occurs in plasmas and in absorbing media near the resonance frequencies, e. g. in silicon dioxide at non-optical bands around 0.05 nm, 6 nm and 20 nm. It does not contradict the theory of relativity which sets the limit to the speed of propagation of signals, while the phase speed describes the motion of wave crests, [8] It is also worth noting that while the refractive index of silicon dioxide is bounded between $n' \approx 0.4$ and $n' \approx 3$, the refractive index of gold spreads from $n' \approx 0.16$ at $\lambda \approx 0.06\,\mu$m to values exceeding $n' \approx 12$ as the wavelength $\lambda > 10$.

The above observations suggest that in silicon dioxide the transitional boundary layer where the effective wave speed of radiation noticeably deviates from c_* represented by (22) values may be as thin as few nanometers in the band characterized by high absorption, but in the two other bands the thickness of this layer could be considerably wider, reaching several microns and going far beyond in the nearly transparent bands, such as around $\lambda \approx 5\,\mu$m. The thickness of the transitional layer in gold is expected to be effectively limited by strong absorption of gold at all wavelengths, except extremely short ones, however, due to the wide range of the refractive index of gold the effective speed of radiation in this layer may vary from about an order magnitude below to an order of magnitude higher than the speed of light of vacuum.

The above expectations for silicon dioxide are confirmed by the top and bottom sub-figures of Fig. 7 for radiation in silicon dioxide in the absorbing band and weakly absorbing spectral bands, respectively. The solid, dashed and dash-dotted lines in the top subfigure correspond to waves with 9.2 μm, 21.3, μm and 10μm wavelengths, respectively, all of which belong to the highly absorbing part of the spectra. Evidently, for radiation in these bands the transitional zone to the universal value is short and does not exceed about ~ 5 nm. The bottom subfigure is noticeably different. The dash-dotted, horizontal line corresponds to radiation with a 5, μm wavelength, which has practically no absorption. The speed of such waves remains constant through the entire body, and it does not approach the universal speed in black bodies. The solid and dashed lines correspond to radiations

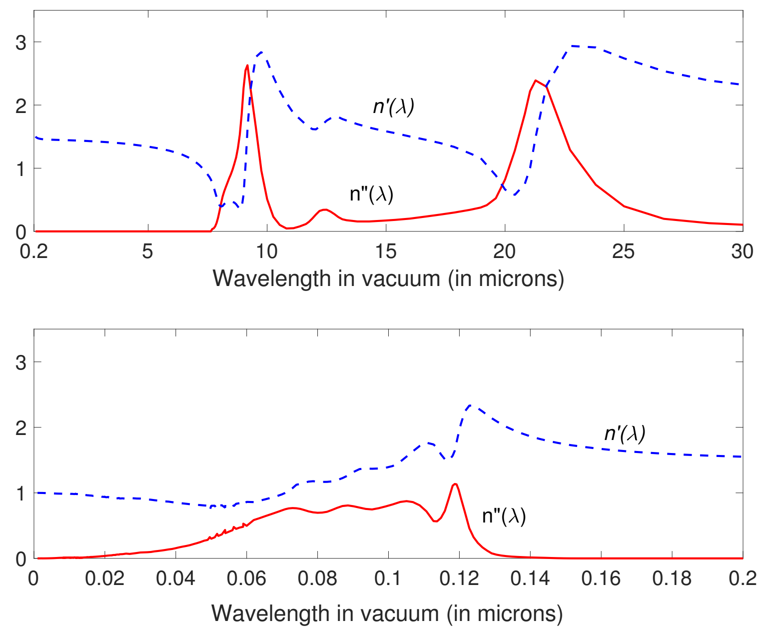


Figure 5: Refractive index $n'(\lambda)$ and extinction coefficient $n''(\lambda)$ of SiO_2

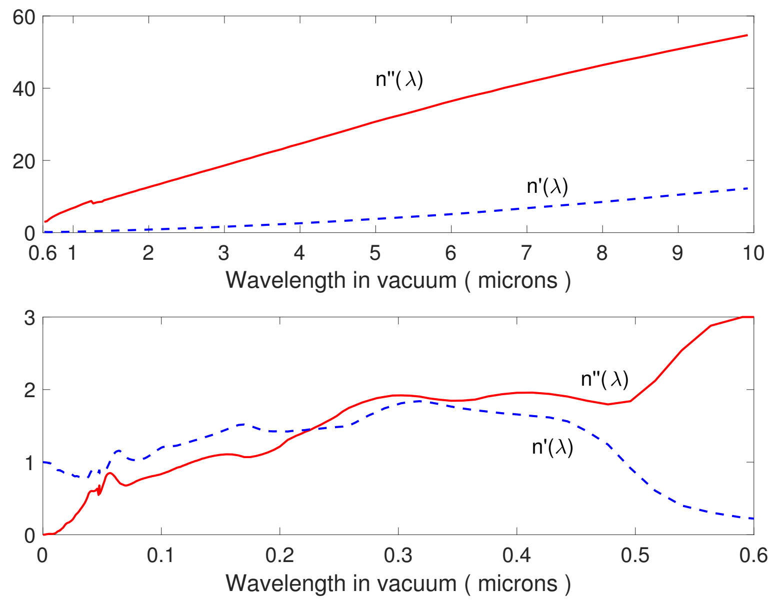


Figure 6: Refractive index $n'(\lambda)$ and extinction coefficient $n''(\lambda)$ of Au

with the wavelengths 0.155μm and 100μm, which have low but still noticeable extinction coefficients. Evidently, for these waves the transitional boundary layers are about 1μm wide, almost two orders of magnitude wider than for radiation in the bands with high absorption.

The dependence of the transitional boundary layer in gold is shown in Fig. 8 and Fig. 9. The five graphs of Fig. 8 correspond to radiation with wavelengths from $3\,\mu$m to $30\,\mu$m, all of which belong to the part of the spectrum where the extinction coefficient and the refractive index monotonically

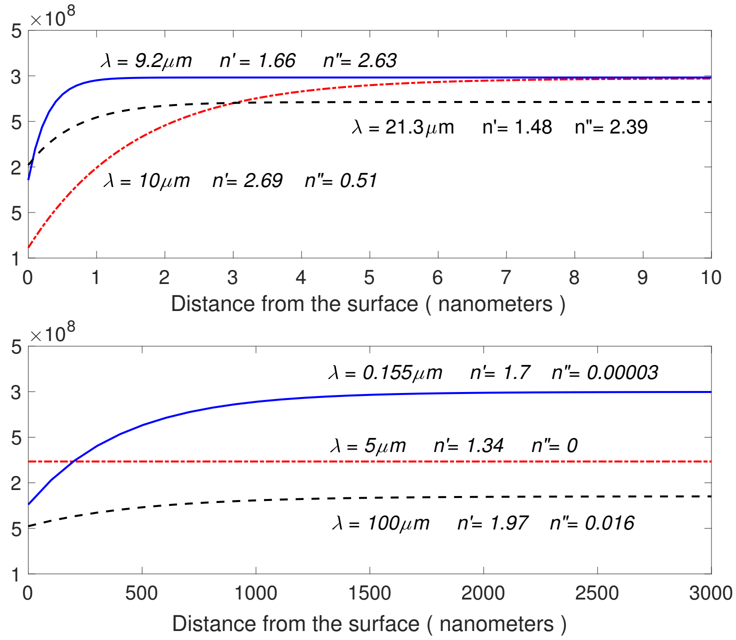


Figure 7: Effective phase speeds near the surface of SiO_2 for different wavelengths

increases with the increase of the wavelength. As expected, all graphs reach the asymptotes corresponding to the universal speed of thermal radiation at the distances from the boundary below 100 nm and reach the conventional values c_0/n' at the surface of the material. Fig. 9 covers a part of the spectra with wavelengths from 600 nm to 1200 nm, where the refractive index of gold dips below unity. Since the absorption in this band remains high, all graphs of the figure approach the universal value c_* within 50 nm from the boundary. However, these graphs approach the asymptote from the above, because due to the small values of the refractive indices the phase speeds of these waves exceed the speed of electromagnetic radiation in vacuum.

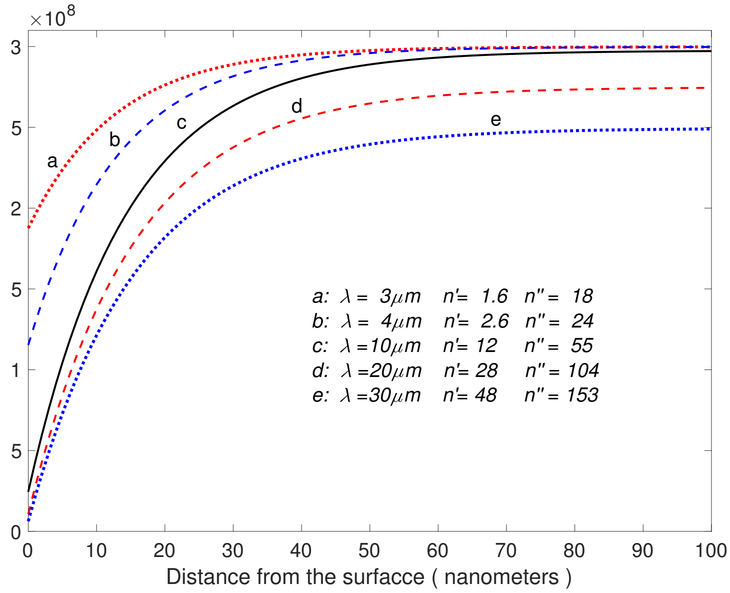


Figure 8: Effective phase speeds near the surface of Au for 1–30 μm wavelengths

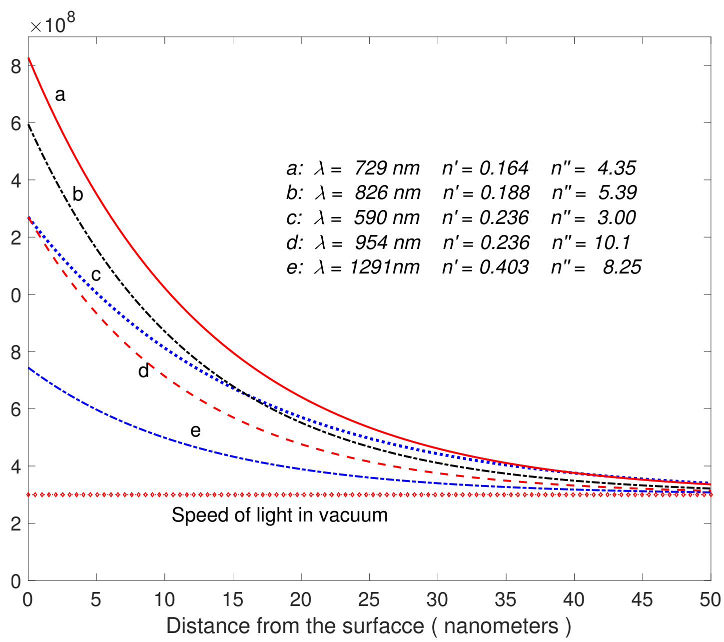


Figure 9: Effective phase speeds near the surface of Au for 0.6–1.2 μm wavelengths

5 Discussion

Thermal radiation sharply differs from other forms of heat transfer by its ability to transfer heat through vacuum and by its stronger than linear dependence on the temperature difference between heat exchanging objects. Due to these features the interest in radiative heat transfer, for almost all time since its inception in the 1700s, [16, 17, 18], has been mostly focused on long-range heat transfer through nearly transparent media, such as heating of the Earth by the Sun, and problems involving very high temperatures, such as combustion or nuclear reactions. These two areas of applications divide the modern theory of thermal radiation on two branches usually referred to as thermal radiation into non-participating and into participating media, respectively.

Thermal radiation in participating media is usually described in terms of the radiative transport theory (RTT) based on the radiative transport equation (RTE), [19, 20, 21], which is similar to the Boltzmann transport equation [22] in the sense that it provides a statistical description of photons considered as particles that move with the speed of light in vacuum between interactions with atoms. This model has been successful in various situations [20, 21] which satisfy a number of conditions formulated in [20, Sec. 9.1.1] with the comment that *"... in the majority of works on the presentation of the RTT fundamentals and application of the theory, the physical suppositions underlying this theoretical presentation are as a rule neither discussed nor analyzed"*. The conditions of applicability of the radiation transport equation include the requirements that the wavelength of the radiation must be essentially shorter than the scale of variations of the structural parameters of the medium. This requirement cannot be relaxed because it is critical for the derivation of the radiative transport equation which represents the balance of the electromagnetic energy in a domain that is much larger than the dominant wavelength of radiation. Besides that, the derivation completely ignores the wave nature of radiation, making it impossible to take into account the interference of waves radiated by different parts of considered structures. Due to theses restrictions, the radiative transport theory cannot be applied to the nanoscale heat transfer despite having been successfully used in a number of other important areas [20, 21].

Since a non-participating medium does not absorb electromagnetic waves, it has a real-valued refractive index, which makes it possible to describe the radiation into such media by the generalization of Planck's law, discussed in Section 1. Correspondingly, the radiative heat transport through non-participating media can be studied by the methods developed for radiation through vacuum. In order to compute the net heat flux between two bodies, these methods first describe radiations from each of these bodies using Planck's law, then the theory of wave propagation is used to compute the rates of energy transmission in each direction, those difference represents the net flux of the radiated energy. This approach has been successful in studying long distance thermal radiation, but it fails in cases when the distance between the bodies is comparable or smaller than the wavelength of the radiation, [23, 24]. However, this failure is not caused by fundamental deficiencies of the approach, but results from its implementation, which needs a few adjustments to take into account specifics of wave phenomena in small scales.

It is well known that electromagnetic waves radiated by sources separated by a long distance compared to their wavelength may be considered as uncorrelated, which means that the energy flux of the superposition of these waves is almost equal to the sum of the fluxes of the individual waves, [3]. This implies that the radiations from two bodies A and B separated by a large distance are independent of each other to the extend that each of them can be considered as if the other one does not exist. Due to this property the radiation from each of the bodies A and B is determined solely by the corresponding temperature T_A or T_B. Evidently, this condition is not met in a system of two bodies separated by a sub-micron distance and maintained at temperatures corresponding to wavelengths exceeding 10 microns, which is the case at room temperature.

In such cases, spectra of radiation from each of the bodies A and B is described by the generalization of the Planck's law to systems with a heat flux, which involves the corresponding temperatures T_A or T_B, as well as the heat flux Q, [25, 26, 27]. Another principal feature of nanoscale systems is that their thermal radiation cannot include electromagnetic waves with certain wave vectors. This happens because electromagnetic fields radiated by different bodies must continue each other to a global field defined in the entire structure. It is shown in [28] that a global field may include only waves whose wave vectors satisfy certain compatibility conditions, which determine "radiative conduction bands", similar to conduction bands in layered semiconductors devices.

If the spectra of thermal radiation in the presence of heat flux and the radiative conductance bands are known, they determine the equation connecting the temperatures of the bodies with the heat flux between them, which makes it possible to compute the flux corresponding to given temperatures. Therefore, since the expressions of the spectra of radiation and the conduction bands are obtained in [27, 28], we have all of the information needed to compute the radiative heat transport between bodies separated by nanoscale gaps. However, the spectra of radiation and the inequalities describing the conduction bands are defined in terms of the speeds of thermally excited electromagnetic waves, which as discussed in Section 2 do not necessarily coincide with the speeds of the signal carrying waves described by a conventional formula in terms of the refractive indices of the media.

6 Summary and Conclusions

We analyzed the concept of the wave speed of thermal radiation and proposed that in absorbing materials it should be described by expressions that take real values and involve the material's temperature, its refractive index and extinction coefficient, as well as the direction of propagation and the distance from the materials surface.

We first argue that Planck's law can not be straightforwardly extended to radiation into an absorbing medium because this law relies on a concept of the speed of light in a matter with a complex valued refractive index that is designed to model signal carrying waves but is not suitable for modeling thermal radiation. Electromagnetic waves which carry signals, such as radio waves, laser beams or radiation from radars, decay in a medium because some of their energies are absorbed. The absorbed energy is eventually re-radiated but these secondary waves appear as noise that is uncorrelated with the original signal.

In order to study thermal radiation in an absorbing material, the material and the radiation must be considered together as a closed system. The energy in such a system is conserved and its distribution between the material and radiation does not change in time. The radiation in such systems admits decomposition into normal modes, which makes it possible to extend Planck's law to radiation into absorbing materials.

The paper proposes a model of thermal radiation coupled with an absorbing medium. This radiation field has normal modes, which correspond to an effective speed of thermal radiation. Assuming an absorbing material and the radiation in it are in thermal equilibrium we show that deep inside the material the average speed of photons is given by a frequency and temperature dependent expression $c_* = c_0/(1 + \mathrm{e}^{-\hbar\omega/\kappa T})$, which does not depend on the material. In an ideal black body this result remains valid everywhere inside the body all the way to its surface, but in realistic materials with finite extinction coefficients, the effective wave speed of thermal radiation depends on the distance from the material surface in a complex way, which involves the distance from the surface, the direction of propagation, as well as the refractive index and the extinction coefficient of the material. The obtained expression gradually converges to expected results in the limiting cases, such as for waves gliding along the surface, perfectly transparent materials, ultra short or ultra long wave radiations, etc.

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