Intense radiative heat transport across a nano-scale gap

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Abstract

In this paper, we analyze the radiative heat transport in layered structures. The analysis is based on our prior description of the spectrum of thermally excited waves in systems with a heat flux. The developed method correctly predicts results for all known special cases for both large and closing gaps. Numerical examples demonstrate the applicability of our approach to the calculation of the radiative heat transport coefficient across various layered structures.

1 Introduction

Here we study the radiative heat transport between two slightly separated parallel plates maintained at temperatures that may differ by hundreds of degrees.

The first studies of this problem were based on the Stefan-Boltzmann law and concluded that the heat flux Q between two black-body half-spaces at temperatures T_A and T_B had the value $Q = \sigma(T_A^4 - T_B^4)$ determined solely by the temperatures and by the constant $\sigma \approx 5.67 \times 10^{-8}\,\mathrm{W/m^2K^4}$, known as the Stefan-Boltzmann constant, [1, 2, 3]. With some corrections, taking into account the deviation of real materials from black bodies, this theory was confirmed by numerous experiments and remained practically unchallenged until the 1960s, when some new experiments demonstrated that as the separation between the plates drops below a few microns, the heat flux significantly increases [4, 5, 6]. Moreover, experiments suggested that as the separation $H \to 0$, the heat flux may diverge at the rate $\sim 1/H^2$. These experiments marked the beginning of studies of micro and nano-scale heat transport.

The first theories of micro and nanoscale radiative heat transport [7] explained the "anomalous" increase of heat transport across narrowing gaps by the contributions of evanescent waves, that, under certain conditions, may be excited and propagate along the interfaces. However, despite explaining some experimental data, these theories were less successful in consistent predictions of the outputs of similar experiments with different geometries and materials. For example, the evanescent wave explanation of the $1/H^2$ divergence of the heat transport between plates is equally applicable to the cases where the plates are made from identical or different materials, but it is known that if the materials are different then the rate of radiative heat transport remains finite even at the limit $H \to 0$. This inconsistency requires effort to avoid a "divergence dilemma at contact in the continuum approach" [8], and it also suggests that this theory may be inadequate.

One flaw of the evanescent wave approach to nanoscale radiative heat transport is discussed in [9], where it is pointed out that this approach systematically employs the Planck law and the Fluctuation-Dissipation theorem [10, 11] beyond their domains of applicability. It is well known that, the Planck law describes the spectrum of thermal radiation from a black body in thermal equilibrium, and the formulation of the Fluctuation-Dissipation theorem also requires full thermal equilibrium in the considered structure. Therefore, these fundamental results can only be applied to the analysis of heat transport in structures that *a priori* do not have any net flux of heat. In particular, the Fluctuation-Dissipation theorem cannot be used to study heat transport unless there is a case-specific justification of its applicability to a non-equilibrium system. As shown in [9], such justification exists in the cases when the separation considerably exceeds the dominant wavelength of thermal radiation, which is $\sim 10\ \mu$m at room temperature. It is also shown in [9], that this justification fails when the separation reduces below the dominant wavelength of thermal radiation.

A self-consistent approach to heat transport by propagating waves was proposed in [12] and then further developed in [13, 14]. In [12] it was applied to the analysis of interface thermal resistance, also widely known as Kapitsa resistance, which is due to acoustic waves, a.k.a. phonons. Then, in [13] this approach was adapted to the study of thermal radiation across a nanoscale vacuum gap, which is the structure whose experimental analysis [15, 16] was hailed as a demonstration of a "breakdown in Planck's law" [17]. More recently the self-consistent approach to nanoscale radiative heat transport was used to estimate the heat transport across vacuum gaps by phonon (i.e. acoustic waves) tunneling and to explain the phenomenon of thermal rectification, which is the asymmetry between heat transport in opposite directions [14].

The previous studies [13, 14] of radiative heat transport across a nanoscale gap were focused on the computations of the heat transport coefficient, which is the ratio $K = Q/\Delta T$ of the net heat flux Q to the infinitesimally small temperature differential ΔT causing this flux. Correspondingly, these studies were restricted to the computation of the net heat flux between plates maintained at nearly equal temperatures. This limiting case gives an insight into the process of thermal radiation, but it does not directly address practical problems arising, for example, in Heat Assistance Magnetic Recording (HAMR) where the temperature differential between the read/write head and the magnetic medium may reach hundreds of degrees and the gap between them is only a few nanometers.

Here we compute the net heat flux between two half-spaces occupied by identical or different materials and maintained at arbitrarily different temperatures. The study shows that as the temperature differential increases to hundreds of degrees, the dependence of the heat flux Q on the temperature differential ΔT undergoes certain qualitative changes, which are discussed. The obtained results have a surprising connection with the concept of temperature in the special theory of relativity and may help to resolve a long standing discussion about the relativistic transformation of the temperature[18, 19, 20, 21].

2 The problem and the representation of its solution

Let two half-spaces be occupied by identical or different media separated by a vacuum gap or some layered structure of materials and maintained at constant temperatures T_A and T_B, as shown in Fig. 1. If the temperatures are different then there is a steady heat-flux Q through the structure. This flux is determined by both temperatures, by the distance between half-spaces H and by the material parameters of the media. We assume that the material is fixed and focus on the analysis of the dependence of the heat flux $Q = Q(T_A, T_B, H)$ on the temperatures and the gap's width.

For convenience we introduce Cartesian coordinates (x, y, z) in such a way that the half spaces with the temperatures T_A and T_B are described by the inequalities $x < 0$ and $x > H$, respectively.

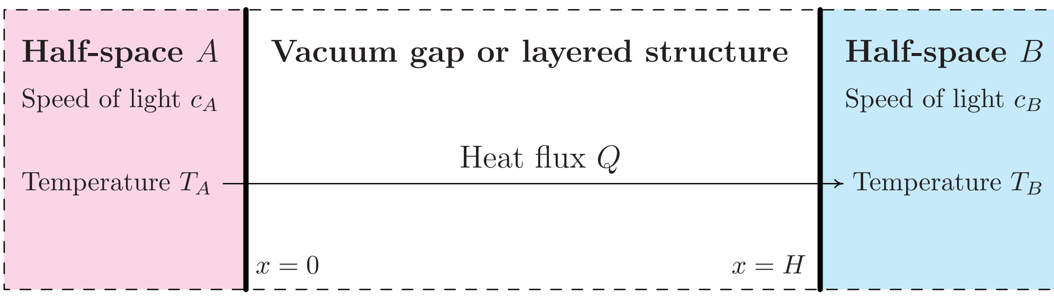

Figure 1: A geometrical sketch of the problem

Following the method proposed in [13] we assume that the net heat flux Q can be represented by the difference

$$Q = Q_{A\to B}(T_A, Q) - Q_{B\to A}(T_B, Q) \tag{1}$$

between the flux $Q_{A\to B}(T_A, Q)$ which passes from the domain $x < 0$ to the domain $x > H$ and is determined by the net flux Q and the temperature T_A, and the flux $Q_{B\to A}(T_B, Q)$ which passes in the opposite direction and depends on the net flux Q and the temperature T_B. Obviously, if the functions $Q_{\alpha\to\beta}(T_\alpha, Q)$ are known then(1) can be treated as an equation with three variables T_A, T_B, and Q, so that if any two of these parameters are known then the third one can be computed.

In order to compute the one-way flux $Q_{\alpha\to\beta}$ coming from the α-th domain we adopt the concept that thermal processes excite in the α-th domain the electric fields

$$\boldsymbol{E}^0_\alpha(\omega, \theta, \phi, \boldsymbol{p}) = a_\alpha(\omega, \theta, \phi)\boldsymbol{p}\, \mathrm{e}^{\mathrm{i}(x\cos\theta + ye_y + ze_z)\omega/c_\alpha - \mathrm{i}\omega t + \mathrm{i}\psi}, \tag{2}$$

where ω is a frequency, $\boldsymbol{e} = (\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$ and $\boldsymbol{p}$ are mutually orthogonal unit vectors, c_α is the phase speed of light in the α-th half-space, $a_\alpha(\omega, \theta, \phi)$ is the amplitude, and ψ is a phase shift. These fields appear as plane waves that propagate along the direction $\boldsymbol{e}$ and are polarized along the unit vector $\boldsymbol{p}$. The parameters ω, $\boldsymbol{e}$ and $\boldsymbol{p}$ of the field (2) are uniformly distributed within their admissible domains, but the directional spectrum $a_\alpha(\omega,\theta,\phi)$ is pre-determined by the thermodynamical characteristics of the medium, which will be discussed in the next section.

The average energy densities of the plane waves (2) of all polarizations with frequencies in the range $(\omega, \omega + \mathrm{d}\omega)$ and with the angular coordinates of the direction of propagation in the ranges $(\theta, \theta + \mathrm{d}\theta)$, $(\phi, \phi + \mathrm{d}\phi)$ have the values

$$\mathrm{d}\mathcal{E}_\alpha(\omega,\theta,\phi) = \frac{1}{4\pi} W(\omega,\theta,\phi) D(\omega, c_\alpha) \sin\theta \mathrm{d}\theta \mathrm{d}\phi \, \mathrm{d}\omega, \tag{3}$$

where

$$W(\omega,\theta,\phi) = \epsilon_\alpha |a_\alpha(\omega,\theta,\phi)|^2 \tag{4}$$

is the time-averaged energy density of the plane waves (2) of all polarizations, $\sin\theta/4\pi$ is the weight needed to perform spherical integration over all directions of propagation of the waves (2), and

$$D(\omega, c) = \frac{\omega^2}{2\pi^2 c^3}, \tag{5}$$

is the density of states defined in such a way that the product $D(\omega,c)\mathrm{d}\omega$ represents the number of normal modes per unit volume of electromagnetic waves of single polarization with the wave speed c and frequencies from the band $(\omega, \omega + \mathrm{d}\omega)$ in a large domain.

Although thermal processes in the α-th medium excite waves (2) no single one of these waves alone produces an electric field in this medium because on the boundary of the α-th domain the wave $\boldsymbol{E}^0_\alpha(\omega,\theta,\boldsymbol{p})$ splits into two parts, one of which is reflected back to the α-th half-space while the other part is transmitted to the opposite half-space. As a result, the total thermally excited electric field in the α-th domain can be represented as a superposition of elementary fields

$$\boldsymbol{E}_\alpha(\omega,\theta,\phi,\boldsymbol{p}) = a_\alpha(\omega,\theta,\phi) \left(\boldsymbol{p} \mathrm{e}^{\mathrm{i}x\cos\theta\omega/c_\alpha} + R_{\boldsymbol{p}}(\omega,\theta,H) \boldsymbol{p}' \mathrm{e}^{-\mathrm{i}x\cos\theta} \right) \mathrm{e}^{\mathrm{i}(ye_y + ze_z)\omega/c_\alpha - \mathrm{i}\omega t} \tag{6}$$

where $R_{\boldsymbol{p}}(\omega,\theta)$ is the reflection coefficient of the wave (2) from the structure, and $\boldsymbol{p}' = (-p_x, p_y, p_z)$ is orthogonal to the direction of propagation $\boldsymbol{e}' = (-\cos\theta, e_y, e_z)$ of the reflected wave.

The time-averaged energy flux carried by the field (6) with all polarizations along the x-axis is represented by the formula

$$S_\alpha(\omega,\theta,\phi) = c_\alpha \cos\theta \, W_\alpha(\omega,\theta,\phi) \big(1 - R^2(\omega,\theta)\big), \tag{7}$$

where $R^2(\omega,\theta) = \big\langle |R_{\boldsymbol{p}}(\omega,\theta)|^2 \big\rangle_{\boldsymbol{p}}$ is the average of $|R_{\boldsymbol{p}}(\omega,\theta)|^2$ computed over all possible polarizations $\boldsymbol{p}$. It is well known that this average reduces to the arithmetic mean

$$R^2(\omega,\theta) = \frac{1}{2}\Big(|R_\perp(\omega,\theta)|^2 + |R_\parallel(\omega,\theta)|^2 \Big) \tag{8}$$

of the squares of two basic coefficients $R_\perp(\omega,\theta)$ and $R_\parallel(\omega,\theta)$, corresponding to electromagnetic waves with so called perpendicular and parallel polarizations, respectively.

The analytic expressions for these reflection coefficients are not discussed here because they are widely available in the literature [22, 23]. Thus, the reflection coefficient of a stack of $(n-1)$ layers of arbitrary thicknesses $h_2, \ldots, h_n$, between two half-spaces, shown in Fig. 2, is represented by explicit formulas

$$R_\nu(\omega, \theta) = \frac{Z_{in}^n - Z_{n+1}}{Z_{in}^n + Z_{n+1}}, \qquad \text{where } \nu \text{ is a polarization,} \tag{9}$$

with "input impedances" $Z_{in}^1, Z_{in}^2, \ldots, Z_{in}^n$ defined by recursive formulas:

$$Z_{in}^1 = Z_1, \qquad Z_{in}^n = Z_n \frac{Z_{in}^{n-1} - iZ_n \tan(\omega h_n \cos\theta_n / c_n)}{Z_n - iZ_{in}^{n-1} \tan(\omega h_n \cos\theta_n / c_n)}, \qquad n > 1, \tag{10}$$

where

$$c_n = \frac{1}{\sqrt{\epsilon_n \mu_n}}, \qquad c_1 \sin\theta_n = c_n \sin\theta_1, \tag{11}$$

and

$$Z_n = \begin{cases} \zeta_n \cos\theta_n, & \text{for the polarization "}\|\text{"}, \\ \zeta_n / \cos\theta_n, & \text{for the polarization "}\perp\text{"}, \end{cases}, \qquad \zeta_n = \sqrt{\frac{\mu_n}{\epsilon_n}}. \tag{12}$$

These expressions are obtained in [22] by the method of generalized impedances resulting in effective recursive expressions.

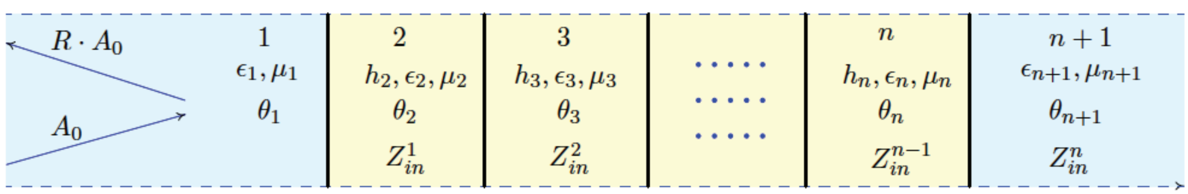

Figure 2: Computation of the reflection coefficient of a stack of layers

In the special case of the structure shown in Fig. 1 with half-spaces from identical materials separated by a vacuum layer with the thickness H, the absolute value of the reflection coefficients (9) are represented by the formulas

$$|R_\nu(\omega, \theta)| = \frac{\left|(Y_\nu^2 - Z_\nu^2)\sin(\gamma H)\right|}{\left|(Y_\nu^2 + Z_\nu^2)\sin(\gamma H) + 2\mathrm{i} Z_\nu Y_\nu \cos(\gamma H)\right|}, \qquad \gamma = \frac{\omega\cos\theta_0}{c_0}, \tag{13}$$

where θ_0 is the incidence angle of the wave in the vacuum layer, Z_ν is the impedance (12) of the incidence wave of polarization ν in the half-space, and Y_ν is the impedance of the corresponding wave in the vacuum layer. Obviously, as $H \to 0$, the reflection coefficients (13) between identical half-spaces approach zero, because $\sin(\gamma H) \to 0$, as H vanishes. However, this property does not hold in a case with different half-spaces, which can be considered by a general method (9)–(12).

Assuming that the amplitude $a_\alpha(\omega, \theta, \phi)$ of (6) is known we see that the total energy flux $Q_{A\to B}$ carried by all waves (6) generated in the medium $x < 0$ and propagating towards the medium $x > H$ can be represented as

$$Q_{A\to B} = \int_0^\infty \int_0^{\pi/2} \int_0^{2\pi} c_A \cos\theta_A \big(1 - R^2(\omega, \theta_A)\big)\, \mathrm{d}\mathcal{E}_A(\omega, \theta_A, \phi_A), \tag{14}$$

Due to the symmetry the expression for the heat flux $Q_{B\to A}$ carried by the radiation excited in the domain $x > H$ and transmitted to the domain $x < 0$ can be obtained from (14) by interchanging indices A and B accompanied by the replacement of θ by $\pi + \theta$. Then, the main equation (1) reduces to the form

$$Q = \int_0^\infty \int_0^{\pi/2} \int_0^{2\pi} \left(1 - R^2(\omega, \theta_A)\right) \left\{ c_A \cos\theta_A \, \mathrm{d}\mathcal{E}_A(\omega, \theta_A, \phi) - c_B \cos\theta_B \, \mathrm{d}\mathcal{E}_B(\omega, \theta_B, \phi) \mathrm{d}\omega \right\} \mathrm{d}\omega, \quad (15)$$

where $\phi_A = \phi_B$, while θ_A and θ_B are related by the Snell law $c_B \sin\theta_A = c_A \sin\theta_B$.

Although the last equation is more specific than (1), it is still not completely specified because the expression (3) for $\mathrm{d}\mathcal{E}_\alpha(\omega, \theta, \phi)$ involves yet indefinite amplitudes $a_\alpha(\omega, \theta, \phi)$. As discussed in [13] we select these amplitudes from the condition that $\epsilon_\alpha |a_\alpha(\omega, \theta, \phi)|^2$ represents the energy spectrum of thermal radiation with a constant heat flux at a certain temperature.

3 The spectrum of thermal radiation in the presence of heat flux

In order to compute the spectrum of thermal radiation in the presence of a constant heat flux, we employ the three-step plan suggested in [24], where it is implemented in the case of an infinitesimally small flux. Assume that the radiation has a heat flux along the x-axis in a reference frame O, and consider it from an auxiliary frame O' that moves along with the heat flux. As the speed of the moving frame increases, starting from zero, the heat flux in this frame decreases, and, eventually, it vanishes at a certain speed V. In the frame with no heat flux, the ensemble of thermally excited waves appears to be in equilibrium, and thus, its energy spectrum can be described by the Planck law of equilibrium thermal radiation [2]. Then, this spectrum can be transformed back to the reference frame O, in which the ensemble has a non-vanishing heat flux.

Consider the electric fields $\boldsymbol{E}^0_\alpha(\omega, \theta, \phi, \boldsymbol{p})$ represented in the reference frame O by the formula (2). This field obeys the equation $\ddot{\boldsymbol{E}}^0_\alpha + \omega^2 \boldsymbol{E}^0_\alpha = 0$, which describes a harmonic oscillator at frequency ω. This implies that the thermal energy density of this wave takes its value from the discrete set $\mathcal{E}_n = \hbar\omega_n$, where $n \geq 0$ is a non-negative integer, which is referred to as the occupational number. In addition, in an auxiliary frame O', which moves relative to O along the x-axis with the speed V, the field $\boldsymbol{E}^0_\alpha(\omega, \theta, \phi, \boldsymbol{p})$ from (2) oscillates with the frequency

$$\omega' = \omega\eta(\theta, V), \quad (16)$$

where

$$\eta(\theta, V) = \frac{1 - V\cos\theta/v}{\sqrt{1 - V^2/c_0^2}}, \quad (17)$$

where c_0 is the speed of light in vacuum and v is the speed of light in the medium. Correspondingly, the thermal energy of this field in the frame O' takes the values $\mathcal{E}'_n = \hbar\omega'_n$.

If the ensemble of waves (2) appears in the frame O' to be in thermal equilibrium at temperature T', then the Planck law implies that the average sum of the occupational numbers of the waves propagating in an arbitrary fixed direction with (θ', ϕ') with its frequency from the interval $(\omega', \omega' + \mathrm{d}\omega')$ has the value

$$\overline{n(\omega', \theta', \phi', T')}\, \mathrm{d}\omega' = \frac{\mathrm{d}\omega'}{\mathrm{e}^{\hbar\omega'/\kappa T'} - 1}, \tag{18}$$

where $\hbar$ and κ are the reduced Planck constant and the Boltzmann constant, respectively. Since the occupational numbers are integers the average sum of occupational numbers of the fields (2) with the frequencies from the interval $(\omega, \omega + \mathrm{d}\omega)$ in the frame O coincides with the average sum of occupational numbers of these fields in the frame O', where their frequencies belong to the interval $(\omega', \omega' + \mathrm{d}\omega')$. This observation leads to the equation $\overline{n}\mathrm{d}\omega = \overline{n'}\mathrm{d}\omega'$, which effectively determines $\overline{n(\omega, \theta, \phi, T)}$ in the reference frame. Indeed, combining this equation with (16) we readily find that

$$n(\omega, \theta, \phi, T) = \frac{\eta(\theta, V)}{\mathrm{e}^{\hbar\omega\eta(\theta,V)/\kappa T'} - 1}. \tag{19}$$

Correspondingly, the average energy density of radiation with frequencies and angles of propagation in the intervals $(\omega, \omega + \mathrm{d}\omega)$, $(\theta, \theta + \mathrm{d}\theta)$ and $(\phi, \phi + \mathrm{d}\phi)$ has the value

$$\mathrm{d}\mathcal{E}(\omega, \theta, \phi) = P(\omega, T; Q)\mathrm{d}\omega\mathrm{d}\theta\mathrm{d}\phi \tag{20}$$

where

$$P(\omega, T; Q) = P(\omega\eta(\theta, V), T') \equiv P\left(\omega\frac{1 - V\cos\theta/v}{\sqrt{1 - V^2/c_0^2}}, T'\right) \tag{21}$$

is the spectrum of thermal radiation from the body at temperature T' in the presence of the net heat flux Q, and

$$P(\omega, T) = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/\kappa T} - 1} \tag{22}$$

is the average thermal energy of a harmonic oscillator at frequency ω in an equilibrium ensemble at temperature T.

The expression (21) for the spectrum of thermal radiation with the net heat flux involves two yet indefinite parameters, which are the speed V of the auxiliary frame O' and the temperature T' in this frame.

If the speed of the moving frame is known then the temperatures in the reference and the moving frames are expected to be connected by an appropriate Lorentz transform. Discussions of this transform started soon after the creation of relativity theory [18, 19, 25] and since then three different transforms have been proposed

$$T' = T\left(1 - \frac{V^2}{c_0^2}\right)^{\nu}, \quad \text{where} \quad \nu = \begin{cases} -1, & \text{proposed in [19]}, \\ 1, & \text{proposed in [20]}, \\ 0, & \text{proposed in [21]}, \end{cases} \tag{23}$$

but none of them has been convincingly explained and accepted. We don't make any judgement of the proposed transform and assume that it is described by the general formula (23) with indefinite ν. This selection does not affect our analysis and it may provide a way to resolve this long standing issue about the relativistic transformation of temperature.

In order to define V, we first apply the condition that in the reference frame the considered ensemble has the energy flux Q. Taking into account that the average thermal energy of the ensemble of the fields (2) is determined by (20) we get the equation

$$\frac{1}{2\pi}\int_\omega\int_\theta\int_\phi cP(\omega\eta(\theta,V),T')D(\omega,c)\cos\theta\sin\theta\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\omega = Q, \tag{24}$$

where the domain of integration must include all waves of the considered statistical ensemble.

In the case when the ensemble includes waves of all frequencies and propagation directions, the substitution $\omega\eta \to \omega$, followed by some calculations, reduces (24) to the form

$$B(V,c)\mathcal{E}(T',c) = Q \tag{25}$$

where

$$B(V,c) = \frac{c}{2}\left(1-\frac{V^2}{c_0^2}\right)^{3/2}\int_0^\pi \frac{\cos\theta\sin\theta\mathrm{d}\theta}{(1-V\cos\theta/c)^3} = V\frac{\left(1-V^2/c_0^2\right)^{3/2}}{(1-V^2/c^2)^2}, \tag{26}$$

and

$$\mathcal{E}(T,c) = 2\int_0^\infty P(\omega,T)D(\omega,c)\mathrm{d}\omega = \frac{(\kappa T)^4}{\pi^2\hbar^3c^3}\int_0^\infty \frac{\xi^3\mathrm{d}\xi}{\mathrm{e}^\xi-1}. \tag{27}$$

Then, recognizing that $\mathcal{E}(T,v)$ represents the energy density of the medium at the temperature T, we convert (24) to the form

$$V\,\frac{(1-V^2/c_0^2)^{3/2}}{(1-V^2/c^2)^2}\left(\frac{T'}{T}\right)^4 = \frac{Q}{\mathcal{E}(T,c)}. \tag{28}$$

Finally, taking into account formulas (23) for the relativistic transform of the temperature we get the equation

$$V\,\frac{(1-V^2/c_0^2)^{3/2+\nu}}{(1-V^2/c^2)^2} = \frac{Q}{\mathcal{E}(T,c)}, \tag{29}$$

which together with (21) determines the power spectrum of the thermal radiation with the steady heat flux Q. It is easy to see that in a non-relativistic case $V \ll c \leq c_0$ the relationship between V and Q reduces to the linear formula $V = Q/\mathcal{E}(T,c)$ which does not depend on the form of the relativistic transform of the temperature.

4 Examples

The usefulness of the obtained results is tested by their applications to layered structures that model those used in the Heat-Assistant Magnetic Recording systems (HAMR).

First we consider a simplest structure where a vacuum layer of the width ranging from $H = 0.1$ nm to $H = 10\,\mu m$ separates two half-spaces of identical materials, one of which is maintained at room temperature $T_B = 273\,°K$ while the temperature of another is up to $\Delta T = 400\,°K$ higher. The wave speed in the half-spaces is set to $0.45c_0$, which makes it possible to compare the results with [13] where the same problem was studied for the cases with a small temperature differential

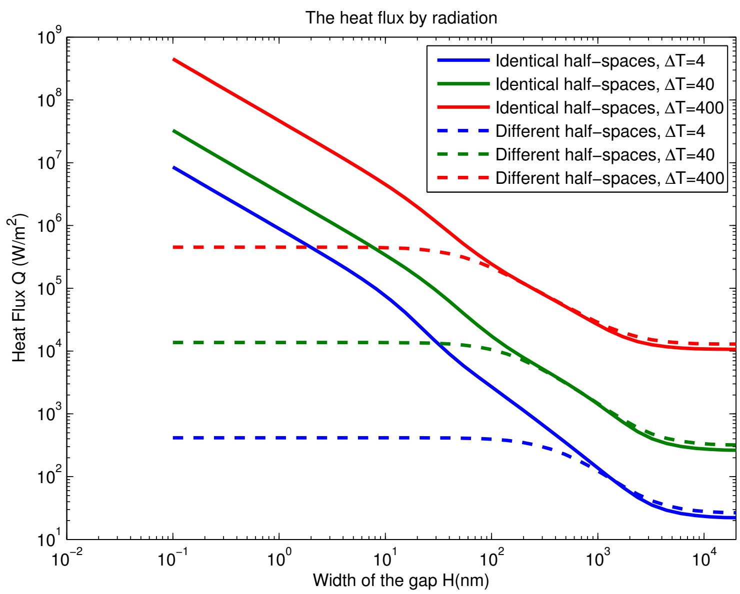

Figure 3: Radiative heat flux between two half-spaces at different temperatures

between half-spaces. The results of the calculations are shown in Fig. 3 by solid lines, which agree with the results from [13] and demonstrate the divergence of the heat flux as $H \to 0$ at the expected rate $\sim 1/H^2$.

Fig. 3 also includes the results of simulation of the radiative heat transport across a vacuum gap between half-spaces from different materials. For better comparison with the previous case, one half-space has the wave speed $c_A = 0.45c_0$, as previously, while the wave speed of the other half-space B is increased to $c_B = 0.65c_0$. The temperature of the half-spaces are set as in the previous case: B is maintained at $T_B = 273\,{}^\circ K$ while the temperature of A is up to $\Delta T = 400\,{}^\circ K$ higher.

The corresponding results are shown by three dashed lines corresponding to three different temperature differentials between the half-spaces. Remarkably, all dashed lines approach horizontal asymptotes in both limiting cases of increasing or decreasing H. As the width of the gap increases, the dashed lines corresponding to heat flux between different materials becomes very close to the solids lines corresponding to heat flux between identical materials. This result agrees with the expectation based on the Stefan-Boltzmann law. As the width of the gap $H \to 0$ becomes noticeably lower than the dominant wavelength of thermal radiation, which is shorter at higher temperatures, the heat flux approaches a constant value.

It is worth mentioning that the behaviors of all graphs in Fig. 3 agree with the concept that waves generally “do not feel objects considerably smaller than their wavelength”. Indeed, when a gap between identical materials becomes narrower than the wavelength, then such gap affects wave propagation as an imaginary interface in a homogeneous material, which must not have any thermal resistance. However, if the materials of the half-spaces are different then a narrow gap affects waves as an interface and the heat flux is determined by the interface thermal resistance, which is independent of H.

Next we consider more complicated structures as shown in Fig. 4 by adding overcoat layers to the surfaces of the half-spaces, maintained at the temperatures $T_B = 273\,°K$ and $T_A = 673\,°K$. The three solid lines in Fig. 4 show the heat fluxes between identical half-spaces coated with 0.5 nm, 1 nm and 2 nm-thick layers from a material with the wave speed $0.7c_0$. These lines noticeably differ from each other only where the gap's width H reduces to a few nanometers and, thus, becomes comparable with the widths of the coating layers. The three dashed lines, all coinciding, in this figure show the heat fluxes between coated half-spaces with wave speeds $c_A = 0.45c_0$ and $c_B = 0.65c_0$, both of which are coated by 0.5 nm, 1 nm and 2 nm-thick layers with the wave speed $0.7c_0$. As the separation H decreases noticeably below the dominant wavelength of thermal radiation these three lines approach the same constant level, which is independent of the thickness of the thin coat. This remarkable behavior, however, fully agrees with the expectation that the layers significantly narrower than the dominant wavelength of radiation cannot noticeably alter the radiative heat transport between the half-spaces.

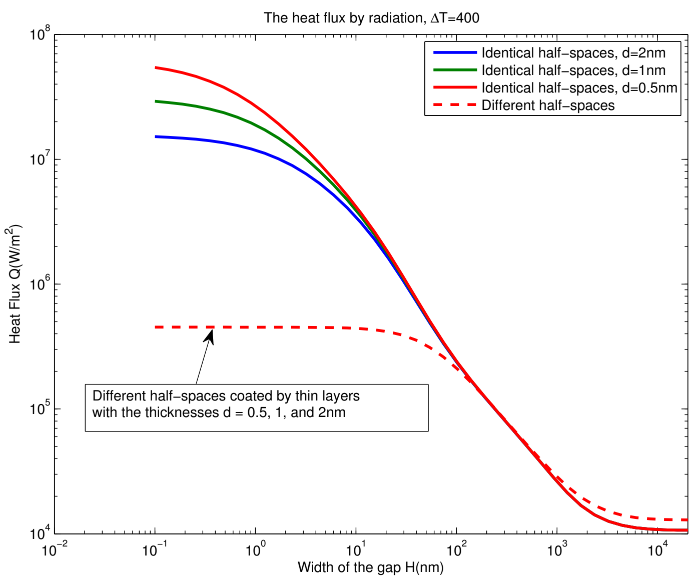

Figure 4: Radiative heat flux between coated and uncoated half-spaces at different temperatures

The obtained results demonstrate that the radiative heat flux across a stack of layers with total thickness below 50 nm is practically independent of the layers. Therefore, for HAMR needs the radiative heat flux across a stack of layers between non-identical half-spaces can be estimated by ignoring the coating layers, which reduces the reflection coefficient of the structure to the value

$$R(\theta)=\begin{cases}\dfrac{\zeta_1\cos\theta_1-\zeta_{n+1}\cos\theta_{n+1}}{\zeta_1\cos\theta_1+\zeta_{n+1}\cos\theta_{n+1}}, & \parallel \text{ polarization},\\ \dfrac{\zeta_1\cos\theta_{n+1}-\zeta_{n+1}\cos\theta_1}{\zeta_1\cos\theta_{n+1}+\zeta_{n+1}\cos\theta_1}, & \perp \text{ polarization}, \quad \zeta_n=\sqrt{\dfrac{\mu_n}{\epsilon_n}}\end{cases} \tag{30}$$

between the substrates without coatings. This expression is much simpler than the general formula (9) for an arbitrary stack of layers, but its main computational advantage comes from its independence of frequency, which simplifies all calculations.

5 Summary

The theory of radiative heat transport across low-micro and nano scale layered structures developed here is based on the assumptions that the spectral distribution of photons in the presence of heat flux is described by the generalization of Planck's law to systems with a non-vanishing heat flux. Then these assumptions are converted by analytic arguments to equations that connect the temperatures on the different sides of the structure with the heat flux across it. Unlike previous studies here we don't apply any restrictions on the thermal differential and the heat flux, which may take arbitrarily large values.

In order to verify the adequacy of the developed theory we used it to calculate the radiative heat flux for several structures similar to those used in Heat Assistant Magnetic Recording (HAMR), which is currently considered as a promising technology for further increase of areal density of data storage. The obtained results are in good agreement with the available experimental data for radiative heat flux. The proposed method also opens a way to study an important HAMR problem of high intensity heat transfer by phonon tunneling across layered structures with possible gaps. All of the above suggests that the proposed method can be further developed into a practical tool, which may be applied to actual structures found in HAMR systems and in other nanoscale devices used for efficient thermal management.

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