

# Observer design for non-uniform sampled systems using gain-scheduling

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## Abstract

In non-uniform sampled systems, the measurements are arriving at irregular time intervals. However, the control is updated at regular time intervals. An observer is required to obtain the estimate of the states during the control update times. We evaluate two observer designs: A Kalman filter and a gain-scheduling observer. The Kalman filter has the optimal performance. However, it is computationally expensive. In contrast, a recent gain-scheduling synthesis technique [1] can be used to design a time varying observer, whose time varying coefficients are a function of the measured sampling time variations. This observer is sub-optimal, but it has significantly less computational complexity as compared to the Kalman filter, which makes it feasible to implement. Simulations are conducted for a self servo writing process in hard disk drives, in order to evaluate performance of  $H_2$  gain-scheduling observer design.

## INTRODUCTION

We consider feedback systems in which the control input is updated at a uniform sampling time, but the feedback measurements are arriving at a nonuniform sampling time[2, 3]. This non-uniformity will be handled by designing an observer, which can estimate system states at a uniform sampling time.

The sampling time variation in the measurement signal can be modeled as plant dynamics variations[4], and the state observer should be designed using this time-varying plant. One approach to deal with this class of dynamics variation is called gain-scheduling. This method explicitly considers plant dynamics variations in the design of time-varying controllers or observers, in order to achieve the defined performance objectives[1]. The performance of this observer can be evaluated by comparing it with that of the Kalman filter[5]. Although the

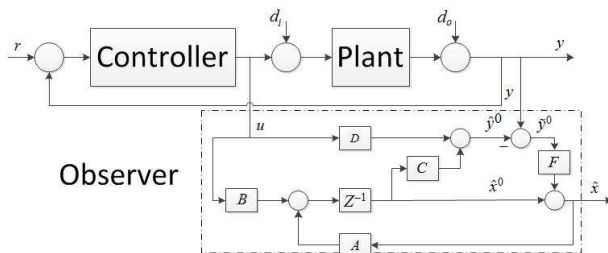


Figure 1: CLOSED LOOP SYSTEM WITH OBSERVER

Kalman filter has optimal performance, observers designed using gain-scheduling techniques are significantly more computationally efficient.

As an example of the systems with nonuniform measurement sampling time, the self servo writing process in hard disk drives[6] is investigated. There are also other sources for the non-uniformity of sampling time[7, 8, 4].

## MODELING

The non-uniformity in the measurement sampling time is considered in the system modeling. The model of this system is defined based on the following assumptions: 1) The measurement can arrive anytime between two consecutive control updates. 2) If no measurement arrives between two consecutive control updates, the estimator will use a priori data as an approximation.

Since the control input is updated at a uniform sampling time ( $T_c$ ), the system states are updated regularly. On the other hand, the output of the system will be at irregular sampling time. This irregularity can be modeled as [4],

$$\begin{aligned} x(k+1) &= A(T_c)x(k) + B(T_c)(u(k) + d_i(k)) \\ y(k) &= CA(\theta_T)x(k) + (CB(\theta_T) + D)(u(k) + d_i(k)) + d_o(k) \end{aligned} \quad (1)$$

where  $\theta_T$  is the time difference between the last control update and the arrival of the new measurement ( $0 \leq \theta_T < T_c$ ). According to Eq.(1), the matrices for state dynamics are constant. However, the matrices for output dynamics are functions of  $\theta_T$ .

## OBSERVER DESIGN

The closed loop block diagram of the system is shown in Fig.1. In this figure, an observer is used to estimate the states by having the control input and measurement signals as its input. This observer is designed based on the following two approaches: Kalman filter and gain-scheduling.

## Kalman Filter Observer

The structure of the Kalman filter is shown in Fig.1. The calculation of gain  $F(k)$  at each time step requires a matrix inversion which is computationally expensive. Despite of its calculation complexity, this filter is proved to be the optimal observer in terms of minimizing the trace of state estimation error covariance. Therefore, this observer will be used to evaluate the performance of the other observer designed using gain-scheduling.

## Gain-Scheduling Observer

The gain-scheduling technique can consider the plant dynamics variation (1) in the design step and obtain a varying observer. The design objective is considered to be the minimization of the state estimation error signal with respect to disturbance signals  $d_i$  and  $d_o$ . Fig.1 shows the structure used for designing both the gain-scheduling observer and the Kalman filter. For the gain-scheduling method, the gain  $F$  is obtained as a function of  $\theta_T$  using the synthesis techniques presented in [1]. This functionality is usually considered to be similar to the functionality of the plant on  $\theta_T$ . For example, If the plant dynamics is a quadratic function of  $\theta_T$ , then  $F(\theta_T)$  will also be a quadratic function.

$$F(\theta_T) = F_2\theta_T^2 + F_1\theta_T + F_0. \quad (2)$$

According to Eq.(2), the gain at each time step can be updated by a few multiplications and summations, which is significantly less computationally expensive as compared to the Kalman filter gain update equations, which require a matrix inversion at each time step.

## SIMULATION RESULTS

The proposed methods for the observer design in systems with non-uniform sampling time is applied to the self servo writing process in hard disk drives, where there is non-uniformity in the measurement sampling time. The model of the VCM is considered to be a simple double integrator. This model is accurate enough, because the self servo writing process is slow and does not excite high frequency dynamics of the VCM. Based on Eq.(1), the dynamics of the system can be written as;

$$A(T_c) = \begin{bmatrix} 1 & T_c \\ 1 & 0 \end{bmatrix}, B(T_c) = \begin{bmatrix} T_c^2/(2J) \\ T_c/J \end{bmatrix}, \quad (3)$$

$$C(\theta_T) = [ 1 \quad \theta_T ], D(\theta_T) = [ \theta_T^2/(2J) ], \quad (4)$$

where  $J$  is the VCM inertia. The performance of the designed gain-scheduling observer is compared with the optimal Kalman filter in terms of trace of covariance matrix for the state estimation error. Also, the observed states are compared with the real values of states available from the computer simulation.

Table 1: THE RELATIVE PERCENTAGE FOR TRACE OF STATE ESTIMATION ERROR COVARIANCE (GAIN-SCHEDULING OBSERVER RELATIVE TO KALMAN FILTER).

		$V_{design}$		
		$10^{-4}$	$10^{-2}$	1
$V_{simulation}$	$10^{-4}$	120 %	75 %	91 %
	$10^{-2}$	173 %	109 %	92 %
	1	186 %	153 %	116 %

The observer design depends on the relative variance of disturbance signals  $D_i$  and  $D_o$ . Therefore, the observers are designed based on different values of the variance  $D_o$  by assuming  $D_i = 1$ . In Table 1, the observers in each column are designed based on a fixed variance of disturbance signals while the simulation are conducted under different variance  $D_o$ . The reason for comparing the performance of detuned observers, in addition to tuned observers, is that in reality the exact variance of disturbance signal may not be known. The results show the relative increase percentage for trace of state estimation error covariance of the gain-scheduling observer, as compared to the Kalman filter. As one can see, the performance of the tuned gain-scheduling observer deviates about 10-20 percent as compared to the tuned Kalman filter. The better performance of the tuned Kalman filter is expected, since it achieves the optimal gain for minimizing the trace of state estimation error covariance. Moreover, the performance of the detuned gain-scheduling observer degrades, as compared to the Kalman filter, by increasing the variance of measurement noise. Based on the simulation results, if the variance of the measurement noise is smaller than its designed value, the detuned gain-scheduling observer achieves better performance than the detuned Kalman filter. The reverse is however true in the case of measurement noise variance being greater than the designed value.

The simulations are run for 5 seconds, but in order to make the plots visible, the results for a small portion of time is sketched in Fig.2. The control update rate is around  $80 \mu s$  and measurements arrival time relative to the latest control update ( $\theta_T$ ) is shown in the first figure. The real values of the states in addition to the observed values obtained by both observers are shown in Fig.2. The observed states can track the real values of states. This tracking is more accurate for the first element of the states.

The main advantage of the gain-scheduling observer, compared to the Kalman filter, is its less computational complexity. The Kalman filter gain requires a matrix inversion at each time step, while the gain for the gain-scheduling observer can be obtained as a polynomial function of sampling time given in Eq.(2).

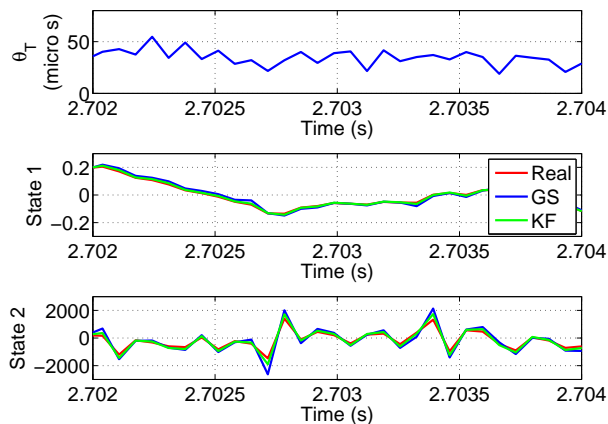


Figure 2: Time domain results.

## CONCLUSION

In this paper, we consider a system in which measurements are arriving at non-uniform sampling times, while the control needs to be updated at an uniform sampling time. This problem is modeled by considering the variation in measurement sampling time as a variation in the plant model. Then, the observer is designed using a gain-scheduling technique which can accommodate this variation in the design stage. The performance of the tuned gain-scheduling observer deviated from the optimal Klamman filter by about 10-20 percent. However, the detuned gain-scheduling observer performs better than the detuned Kalman filter for the case that the measurement noise variance is smaller than its designed value. It's worth mentioning that the main advantage of the gain-scheduling observer is its less calculation complexity as compared to the Kalman filter. The gain for the gain-scheduling observer is a polynomial function of sampling time which makes it feasible to implement.

## References

- [1] De Caigny, J., Camino, J. F., Oliveira, R. C., Peres, P. L. D., and Swevers, J., 2010. "Gain-scheduled h2 and hinf control of discrete-time polytopic time-varying systems". *IET control theory & applications*, **4**(3), pp. 362–380.
- [2] Yu, M., Wang, L., and Chu, T., 2005. "Sampled-data stabilisation of networked control systems with nonlinearity". In *Control Theory and Applications*, IEE Proceedings-, Vol. 152, IET, pp. 609–614.

- [3] Semba, T., Kagami, N., and Tokizono, A., 2001. Method and apparatus for suppressing mechanical resonance in a disk drive storage device using a notch filter. US Patent 6,219,196.
- [4] Shahsavari, B., Conway, R., Keikha, E., Zhang, F., and Horowitz, R., 2013. “Robust track-following controller design for hard disk drives with irregular sampling”. *Magnetics, IEEE Transactions on*, **49**(6), pp. 2798–2804.
- [5] Kalman, R. E., 1960. “A new approach to linear filtering and prediction problems”. *Journal of basic Engineering*, **82**(1).
- [6] Cloke, R. L., 2005. Disk drive for circular self servo writing and spiral self servo writing using prewritten reference servo bursts. US Patent 6,977,789.
- [7] Guo, G., and Yu, J., 2010. “Servo format trend and servo channel improvement for high areal density”. In *APMRC, 2010 Digest*, IEEE, pp. 1–2.
- [8] Ehrlich, R. M., 2005. Methods for improving servo-demodulation robustness. US Patent 6,943,981.