On thermal radiation across nanoscale gaps

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Abstract

Current Heat Assistance Magnetic Recording (HAMR) hard disk drive systems are designed to heat a 20 nm writing spot on the disk to about 400°C by a Near Field Transducer (NFT) located within the slider and spaced a few nanometers from the disk. It is important for the NFT to remain relatively cool to insure that it will not fail during the required lifetime of the drive. Due to the nanometer scale separation between the heated spot and the NFT, the heat exchange between them cannot be accurately calculated by conventional methods. This paper first explains why the conventional models of heat transfer fail at the nanoscale. Then we estimate the heat radiation and heat conduction due to phonon tunneling using an extension of Planck's law from equilibrium systems to systems with a non-vanishing heat flux [2]. It is shown that the heat transport across a few nanometers wide gap is expected to be high enough to provide significant back-heating of the NFT from the heated spot on the disk.

1 Introduction

The data density of hard drives is rapidly approaching a limit after which further increase cannot be achieved without radical modifications of the design that has been essentially evolutionary since 1956, when IBM unveiled its first HDD.

A significant change occurred about a decade ago by moving from the traditional parallel recording to perpendicular recording. However, this change alone is not sufficient for reaching the projected new levels of 10 Tb per square inch. It is expected that further increase in the data density of hard drives will require that a bit of recorded information occupy a spot on the disk smaller than 20 nm in diameter, and that this spot must get some form of energy assistance to lower the medium's coercivity. In Heat Assistance Magnetic Recording (HAMR) systems such assistance is provided by local heating by a laser source of the magnetic medium to near its Curie temperature of about 400°C. Therefore, any HAMR system inevitably includes a closely separated disk and read/write head with different temperatures, and, consequently, the design of such systems must include the analysis of heat exchange between bodies at different temperatures separated by a few nanometers.

There are several mechanisms of heat transport between bodies separated by a narrow air-gap. The heat can be carried by electromagnetic radiation, by "phonon tunneling" caused by intermolecular interactions across the gap, and heat conduction through the air enhanced by convection. The first two of these mechanisms are associated with the processes of electromagnetic and acoustic wave propagation, respectively, and, thus, can be studied by similar methods. Heat radiation across nanoscale gaps was studied in [1] by a novel method based on the extension of the Planck's law for equilibrium systems to systems with a steady heat flux [2]. This method can also be applied to the analysis of heat transport by acoustic waves, i. e. by phonon tunneling, and here we focus on this method and its implications for the design of HAMR systems.

Current designs of HAMR systems employ lasers to heat the required $\sim$ 20 nm spot on a disk. Since such lasers produce radiation with the wavelength of $> 500\,\mathrm{nm}$ the spot size is far below the diffraction limit so it is not possible to use an optical lens to focus the light. To go around this difficulty, currently designed HAMR systems focus laser radiation using a Near Field Transducers (NFT). Such transducers couple light from a laser to plasmonic oscillations, which are the oscillations of the electromagnetic field coupled with electrons in the material. Plasmonic oscillations have a wavelength short enough to be focused down to $\sim 20\,\mathrm{nm}$ at the tip of the NFT, located within a few nanometers from the disk.

The design of a light delivery and heating system using a plasmonic NFT must take into account multi-physics phenomena, including the distribution of electromagnetic fields, their interaction with electrons in the materials of the NFT and of the disk, as well as heat exchange between the NFT and the disk. Such complex problems are often solved by NFT designers by using commercial codes, such as COMSOL or ANSYS, to design structures that, according to the solutions obtained, heat the writing spot on the disk to 400°C while keeping the NFT at a relatively cool temperature below 100°C.

Despite the promising results of multi physics numerical simulations of the light delivery structures, the prototypes of HAMR drives are reported to have short lifetimes because of NFT failure, which is not acceptable for real devices. The causes of NFT failure are not yet identified, but overheating is considered to be a major factor. Therefore, it is necessary to analyze every step of the NFT modeling and identify possible flaws of the employed techniques.

2 Shortcomings of the conventional approaches for nanoscale heat transfer

The theory of heat transfer considers three means of heat transport: conduction, radiation and convection, each of which is governed by its own mathematical model.

Heat convection in HAMR systems may play a role because of the airflow between the NFT and the disk, but it is not expected to be the major cause of NFT heating, so here we focus on the other two mechanisms, which are not associated with mass flow.

Heat conduction is observed in most materials and follows the Fourier law $\boldsymbol{Q} = -\kappa \boldsymbol{\nabla} T$, which states that the heat flux $\boldsymbol{Q}$ is negatively proportional to the gradient of the temperature T. The coefficient κ is referred to as the thermal conductivity, and it is considered to be a basic material parameter. The Fourier Law implies that the time evolution of the temperature distribution in a spacial domain is governed by the heat/diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \qquad \alpha = \frac{\kappa}{c_p \rho}, \tag{1}$$

where α is the "thermal diffusivity", ρ is the mass density and c_p is the specific heat of the material.

Heat radiation is normally considered as the only means of heat transport in vacuum, and it may also dominate heat transport through gases. The conventional theory of thermal radiation is based on Planck's law which states that the density of radiation of a single polarization from a black body at temperature T in the directions characterized by the spherical angles from the intervals $(\theta, \theta + \mathrm{d}\theta)$ and $(\phi, \phi + \mathrm{d}\phi)$ has the power spectrum

$$\mathcal{E}(T, \omega) = \frac{1}{4\pi}\Theta(\omega, T)D(\omega), \tag{2}$$

$$\Theta(\omega, T) = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/kT} - 1}, \quad D(\omega) = \frac{\omega^2}{2\pi^2 c^3}, \tag{3}$$

where $\hbar$ and k are the Planck and Boltzmann constants.

Planck's law makes it possible to estimate the radiation of a "black body" half-space $x < 0$ maintained at temperature T into the half-space $x > 0$ as

$$\int_0^\infty \int_0^{\pi/2} \mathcal{E}(T, \omega)\, c \cos\theta \sin\theta \,\mathrm{d}\theta \mathrm{d}\omega = \sigma T^4, \tag{4}$$

where σ is a constant. Then, the heat flux between two half-spaces maintained at temperatures T_A and T_B is computed by the Stefan-Boltzmann formula

$$Q = \sigma(T_B^4 - T_A^4), \tag{5}$$

which does not involve the distance H between the half-spaces, so according to this formula H could be millions of miles or one nanometer.

The outlined theories of heat conduction and radiation have been verified by an enormous amount of experiments dealing with heat exchange in high-micrometer and larger scales. However, these theories do not agree with measurements in low-micrometer and nanoscale domains. These failures are often considered as surprising, but a closer look reveals that they are inevitable because the conventional theories implicitly assume that all characteristic dimensions, such as the sizes of heat exchanging bodies and the distances between them, are limited from below by certain thresholds.

The heat equation (1) is also known as the diffusion equation because it describes the diffusion of randomly moving particles which interact with each other by collisions. This coincidence suggests that heat conduction may be caused by diffusion of some carriers, and this suggestion is supported by the fact that a phenomenological Fourier law is actually derivable from basic principles only in the cases when the heat transport is modeled by the random motion of some "particles". In gases the Fourier law is provided by a random motion of gas molecules, while in conducting solids it is provided by the random motion of free electrons, and, in dielectric solids, it is provided by a random motion of wave-packets formed by acoustic waves, which are considered as quasi-particles called "phonons", so that the acoustic field can be treated as a gas of phonons.

The kinetic theory of heat conduction explained by diffusion of some carriers can only be applied in domains that are considerably larger than the carriers and their mean free paths between collisions. This, obviously, sets limitations on the applicability of the theory of heat conduction based on the Fourier law.

Since the size of and the mean free path of air molecules are about a few ångstroms and ∼65 nm, respectively, the heat conduction in air may not be expected to follow the Fourier Law and the heat equation in domains smaller than, at least, a hundred nanometers in all directions.

To estimate the applicability of the Fourier Law to heat conduction by phonons we observe that the wavelengths of thermally excited acoustic vibrations in solids at room temperature are of the order of a couple of nanometers. On the other hand, the size of a wave packet is always considerably larger that the dominant wavelength of the involved waves. The above implies that wave packets formed by thermally excited acoustic vibrations have the size of the order of ∼10 nm and higher, so that such quasi-particles, which are often incorrectly called "phonons", can diffuse only in domains not smaller than several tens of nanometers in all directions. This estimate agrees well with numerous experimental reports that the heat transport in nanostructures start deviating from the Fourier Law when one of the structure dimensions falls below 100 nm, [3].

We also observe that as the scale is reduced, the radiative heat transport must also start deviating from the classical theory.

Indeed, at moderate temperatures, below a thousand degrees Kelvin, heat radiation is dominated by electromagnetic waves with wavelengths of several hundreds of nanometers. Therefore, general principles suggest that properties of such waves in domains smaller than a few microns cannot be similar to their properties in larger domains.

In order to illustrate this general principle consider the Stefan-Boltzmann formula (5) which describes radiative heat transport between to two half spaces A and B made from identical materials separated by a vacuum gap. This formula assumes that the half-spaces are maintained by thermostats at local equilibriums at well-defined temperatures T_A and T_B. However, the concept of local equilibriums in the bodies maintained at different temperatures and separated by a nanoscale gap may not be defined at all. Indeed, if the distance between the bodies reduces below the dominant wavelength of heat radiation then, as the separation approaches zero, the influence of such a gap must vanish, so that thermal radiation propagates as in a single uniform medium. But a single uniform medium cannot be in thermal equilibrium with different temperatures in the different parts. Since the collapse of two media to a single one cannot occur abruptly, it is clear that the classical theory of heat radiation, including the Stefan-Bolzmann formula, must start failing as the separation between bodies reduces below a few microns.

The failures of the conventional approach to heat transport at the nanoscale were noticed back in the 1960s when [4] reported an unexpectedly high heat transport coefficient between bodies separated by less than a few microns. The first explanations of these experiments were soon developed into the now conventional approach to radiative heat transport across micro and nanoscale gaps [5]. The predictions of this approach could be made to match the experiments from [4], but it needed to incorporate specific assumptions about the sources of thermally excited radiation, which undermined the predictive capability of this theory.

After more than four decades since the publication of [5], the theory of nanoscale radiative heat transport remains unsettled, which suggests that this theory may have flaws in its foundations, which show up only in nanoscale structures.

To find the flaw in the the contemporary approach to radiative heat transport we start from the observation that it assumes that thermal radiations from the bodies are additive, so that the process of heat exchange is calculated as described in [5, Sec.V]:

"We first consider the fields set up by the noise sources in the medium at $z < 0$ and calculate the (averaged) Poynting vector in the z direction at the point d_+, i.e. just inside the second medium. This is the heat transferred to the second medium due to thermal radiation from the first. In the same way we calculate the Poynting vector in the $-z$ direction at $z = 0_-$, due to sources in the second medium. The difference of the two expressions is the net energy transfer due to the temperature difference of the two bodies."

and in [6, Sec.2]

"Since the sources of the thermal fluctuations in media 1 and 2 are statistically independent, both fields are incoherent, and it is this which causes the additivity of fluxes ... ",

These assumptions imply that the net heat flux between bodies A and B can be represented as

$$Q = Q_{A\to B}(T_A) - Q_{B\to A}(T_B), \tag{6}$$

where $Q_{A\to B}(T_A)$ is the flux originated in A, determined by T_A and delivered to B, while $Q_{B\to A}(T_B)$ is the flux originated in B, determined by T_B and delivered to A. Then, the analysis of heat transport between A and B reduces to the computations of thermal radiations from each of the domains A and B, which is considered as a routine problem.

However, it is easy to see that there is a flaw in the reasoning leading to (6). Indeed, consider the assumption about maintaining heat exchanging bodies at local equilibrium at different temperatures. If the bodies A and B exchange heat then these bodies interact. Therefore, as explained in the very beginning of [7, Sec.1-1], the interacting bodies A and B must be considered together as a single thermodynamical system. In this case, if the system's parts are in local equilibriums, then the entire structure is in equilibrium. This means the temperature must be uniform over the entire system and the net heat flux must vanish everywhere in the system. Therefore, $T_A = T_B$ and there is no heat transport between A and B.

Despite the transparency and generality of the above reasoning there is a common misconception [5] that the net heat flux between the bodies A and B can be represented by (6), and that it can be rigorously justified by the Fluctuation-Dissipation theorem, which appears as the centerpiece of the description of thermal radiation in terms of Fluctuational Electrodynamics [7, 8, 9].

Fluctuational Electrodynamics is a phenomenological theory based on an assumption that thermal radiation from a domain V filled by an absorbing material is generated by stochastic extraneous currents $\boldsymbol{J}(\boldsymbol{x},\omega)$ and $\boldsymbol{J}(\boldsymbol{y},\omega)$ whose Cartesian components $J_p(\boldsymbol{x},\omega)$ and $J_q(\boldsymbol{y},\omega)$ satisfy the Fluctuation-Dissipation theorem [8, 9], which states that **if the domain V is maintained at thermal equilibrium at uniform temperature T, then for all $x \in V$, $y \in V$**

$$\left\langle J_p(\boldsymbol{x},\omega)\overline{J_q(\boldsymbol{y},\omega)}\right\rangle = C\omega\epsilon''(\omega)\Theta(\omega,T)\delta_{pq}\delta(\boldsymbol{x}-\boldsymbol{y}), \tag{7}$$

where $\epsilon''(\omega)$ is the imaginary part of the relative dielectric parameter of the medium, C is a universal constant, δ_{pq} and $\delta(\boldsymbol{x})$ are the standard δ-functions.

However, this theorem explicitly requires that the temperature be constant over the entire considered structure, and, thus, this theorem can only be used in the situations when *"... the role of the transport phenomena is as yet insignificant"*, [8, Page 112], i. e. in situations when the net heat transport vanishes *a priori*.

3 Extensional of the Planck's law to systems with a net heat flux

The above discussion shows that the conventional theory of heat radiation employs the concept of *"local thermal equilibrium"* which implicitly assumes that the separation between "local" domains A and B is sufficiently large to consider these domains as not interacting with each other, so that the spectra of radiations from A and B may be computed by the Planck's formula, which is limited to closed systems in thermal equilibrium. Therefore, this theory is ultimately inconsistent because it represents a non-vanishing net flux using the formula which is valid only when flux vanishes. In some cases such inconsistency may be tolerable and lead to rather accurate approximations. However, an inconsistent theory may not always produce acceptable results, so that its persistent disagreements with experiments is not surprising and should stimulate effort to eliminate the major inconsistencies of the theory.

The observation that the conventional theory computes a non-vanishing heat flux using the Planck's law which is valid only when this flux vanishes, suggests that extension of the theory of radiative heat transport can be improved by the extension of the Planck's law from equilibrium systems to systems with a non-vanishing net heat flux.

The required extension of the Planck's law is derived in [2], where it is shown that if there is a net heat flux $\boldsymbol{Q} \neq 0$ then the radiation from a black body is not isotropic, as in the equilibrium case, but depends on the direction of propagation.

More precisely, the results of [2] imply that if the ensemble of thermally excited waves has the net flux $|\boldsymbol{Q}| \ll 1$, then the average energy density of a single wave from this ensemble propagating along the unit vector $\boldsymbol{e}$ has the power spectrum

$$\Theta(\omega, T_A; Q, \theta) = \Theta\left(\frac{\omega}{1 + \boldsymbol{Q} \cdot \boldsymbol{e}/cE}, T\right), \tag{8}$$

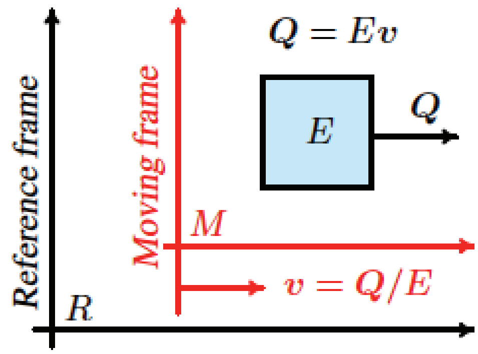
where c is the wave speed, ω is the frequency, $\Theta(\omega, T)$ is the Planck function (3), and E is the energy density of thermally excited acoustic waves of all polarizations, all frequencies and all directions of propagation.

To derive (8) it suffices to consider the system in an auxiliary frame that moves with the speed $\boldsymbol{v} = \boldsymbol{Q}/E$ relative to the reference frame, where the system has the net flux $\boldsymbol{Q}$, as shown in Fig. 1. In the moving frame the system has no net flux, which implies that the average energy of a single wave is described by the Planck's function $\Theta(\omega, T)$ from (3). Then, applying the Doppler transform we return to the reference frame and get (8).

4 Phonon tunneling across nanoscale gaps

The extension of the Planck's law to systems with a non-vanishing heat flux provides a means for routine calculations of the heat transport coefficient caused by propagating waves, including but not limited to both electromagnetic and acoustic waves.

To illustrate the power of the proposed technique we first consider heat transport by acoustic waves, i. e. by phonon tunneling, between two half-spaces $x < 0$ and $x > H$ occupied by identical materials.



If Q *is the flux in the frame* R*, then in the moving frame* M *with* $v = Q/E$ *there is no flux and radiation has Planck's spectrum. Spectra in* M *and* R *are related by Doppler shift*

Figure 1: Modification of the Planck Law for systems with a heat flux

To demonstrate how acoustic waves can carry heat across a gap, it suffices to consider the model shown in Fig. 2 where two material half-spaces $x < 0$ and $x > H$ are separated by a vacuum gap $0 < x < H$, whose faces are maintained near the equilibrium positions $x = 0$ and $x = H$ by some external force that depends on the current position of the faces. Due to thermally excited lattice vibrations in the half-spaces, their faces experience the displacements of lattice vibrations making the separation between the faces not exactly the constant value H. On the other hand, since intermolecular forces have a rather long range relative to the atomic spacing, the motion of the atoms near the surface in a body may exert time dependent forces on molecules at distances as far as several microns outside the body [10]. Therefore, lattice vibrations in slightly separated bodies interact across the gap and thus can transfer energy between bodies.

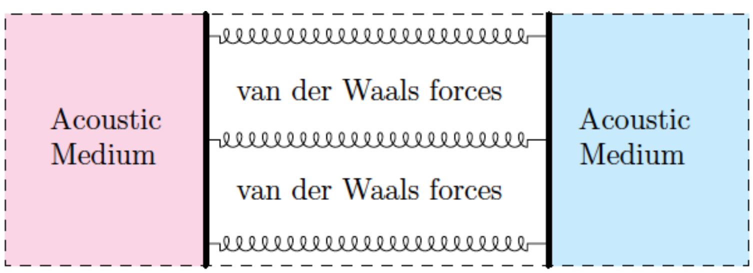


Figure 2: Interaction of two acoustic media separated by narrow gap

To develop the outlined concept of thermal interaction between separated bodies and obtain a quantitative description, it is necessary to adopt some simplifying assumptions about the lattice vibrations. First, we assume that the materials are isotropic, homogeneous and can be described by a linear theory of elasticity that considers the averaged characteristics of atomic motion computed over microscopic domains containing sufficiently large numbers of atoms. This theory implies the existences of three types of elastic waves: longitudinal waves propagating with the speed c_p and two kinds of transverse waves with the lower speed c_s. Waves of different polarizations propagate inside the media independently of each other, but they strongly interact at the boundaries causing significant complications of the total picture of wave propagation. To eliminate difficulties caused by the existence of the different kinds of elastic waves, it is common to describe the thermal properties of solids in terms of the simpler Debye model, which assumes that all three types of thermally excited waves are entirely independent of each other, have their frequencies in the band $0 < \omega < \omega_{\rm D} \equiv \kappa T_D/\hbar$, where T_D is the Debye temperature, considered as a material parameter, and they propagate as acoustic waves in a compressible fluid or gas with the single wave speed c determined by the equation $1/c^3 = (1/c_p^3 + 2/c_s^3)/3$.

It is well known that acoustic waves with the speed c can be described in terms of a scalar velocity potential ψ that satisfies the wave equation $\ddot{\psi} = c^2\nabla^2\psi$, and defines the pressure p and the acoustic displacement vector $\boldsymbol{\xi}$ by the formulas $p = -\rho_0\dot{\psi}$, and $\dot{\boldsymbol{\xi}} = \boldsymbol{\nabla}\,\psi$, where ρ_0 is the density of the medium in the unperturbed state [11].

Let the oscillations of the media depend on time by the exponential factor $\mathrm{e}^{\mathrm{i}\omega t}$. Then the oscillations in the half spaces are described by the Helmholtz equations

$$\nabla^2\psi + \frac{\omega^2}{c^2}\psi = 0, \qquad x < 0, \quad x > H, \tag{9}$$

where c_D is the sound speed in the half-spaces and ψ is the velocity potential which determines the pressure p and the displacement $\xi(x,y,z)$ along the x-axis by the formulas

$$p = -\mathrm{i}\omega\rho\psi, \qquad \xi = \frac{\psi'_x}{\mathrm{i}\omega}, \qquad \ddot{\xi} = \mathrm{i}\omega\psi'_x. \tag{10}$$

Since the van der Waals forces between the open faces of the bodies $x < 0$ and $x > H$ depend on the displacements of these faces, the Helmholtz equations (9) are not complete and must be supplemented by the interface conditions

$$p(x,y,z) = F(H, \xi(x,y,z)) = p(x+H,y,z), \tag{11}$$

where $F(H, \xi(x), \xi(x+H))$ is the van der Waals force between two half-spaces separated by distance $D = H + \xi(x+H) - \xi(x)$.

Let an incident wave $\mathrm{e}^{\mathrm{i}\omega(x\cos\theta + y\cos\theta_y + z\cos\theta_z)/c}$ propagate in the domain $x < 0$. Due to the interaction of this wave with the boundaries $x = 0$ and $x = H$, it generates the reflected wave

$$R\mathrm{e}^{\mathrm{i}\omega(-x\cos\theta + y\cos\theta_y + z\cos\theta_z)/c}$$

propagating in the half-space $x < 0$, and transmitted wave

$$K\mathrm{e}^{\mathrm{i}\omega((x-H)\cos\theta + y\cos\theta_y + z\cos\theta_z)/c}$$

propagating in the half-space $x > H$. As follows from [1], in order to estimate the coefficient of heat transport by acoustic waves across a vacuum gap it suffices to compute the reflection coefficient R of the plane wave with an arbitrary incident angle θ.

To compute the reflection coefficient determined by the equations (9) and (11) we approximate the van der Waals forces by the expression from [10] and get the result

$$R = \frac{-\omega\rho}{\omega\rho + 2\mathrm{i}\alpha}, \quad \alpha = \frac{A}{H^2 c}\left(\frac{\omega^2 \sin^2\theta}{2c^2} - \frac{1}{H^2}\right), \tag{12}$$

where A is the Hamaker constant with a typical value of about 10^{-19} J, [10, Sec 11.1].

It is easy to see that at the limit $H \to \infty$ we have $\alpha \to 0$ and $R \to -1$, which agrees with the expectation that wide gaps reflect all incoming waves. In the opposite limit $H \to 0$, we have $\alpha \to \infty$ and $R \to 0$, which agrees with the expectation that a vanishing gap between identical materials does not reflect. In the general case R in (12) is bounded as $|R| \leq 1$ and must be computed numerically.

Let the half spaces A and B be maintained at the temperatures T_A and T_B, respectively. If $T_A \neq T_B$ then there is a non-vanishing net heat flux $Q = Q(H, T_A, T_B)$ carried by acoustic waves and determined by T_A and T_B, by the width H, and by the material of the half-spaces. The heat conduction coefficient of the gap is defined as

$$K(H; T_B) \approx \frac{Q(H, T_A, T_B)}{T_A - T_B}, \quad T_A \to T_B. \tag{13}$$

As shown above, heat conductance across a narrow gap can be described by wave equations and interface conditions that are similar to those arising in the analysis of heat radiation [1]. Therefore, in order to estimate the heat conductance coefficient of a gap it suffices to adopt the method developed in [1] and proceed along the following steps: 1) assume that T_A and the net flux Q are known and compute the heat flux $Q_{A\to B}(T_A, Q)$ from A to B; 2) assume that T_B and Q are known and compute the heat flux $Q_{B\to A}(T_B, Q)$ from B to A; 3) write the equation

$$Q = Q_{A\to B}(T_A, Q) - Q_{B\to A}(T_B, Q), \tag{14}$$

and solve it for Q, assuming that T_A and T_B are known.

It is shown in [2] that if there is a net heat flux $Q \neq 0$ then the conduction of thermally excited acoustic waves inside A and B is not isotropic, but depends on the direction of propagation. More precisely, the results of [2] imply that if $Q \ll 1$, then inside A a thermally excited acoustic wave of a single polarization propagating in the direction characterized by the spherical angles (θ, ϕ) has the power spectrum

$$\mathcal{E}(\omega, T_A, Q, \theta) = \Theta\left(\frac{\omega}{1 + q_A \cos\theta}, T_A\right)\frac{D(\omega)}{4\pi}, \tag{15}$$

where $q_A = Q/cE_A$, c is the wave speed in the half-spaces, and

$$E_A = \frac{3}{2}\int_0^{\omega_\mathrm{D}} \mathrm{d}\omega \int_0^{\pi} \mathcal{E}(\omega, T_A, Q, \theta)\sin\theta \mathrm{d}\theta \tag{16}$$

is the energy density of thermally excited acoustic waves of all three polarizations (accounted for by the factor "3"), of all frequencies (accounted for by the integration over ω through the cut-off Debye frequency ω_D) and radiated in all directions (accounted for by the integration over θ). A small heat flux $Q \ll 1$ affects (16) only in the second order, so that in the first order approximation E_A is determined solely by the temperature of the medium.

In order to study the heat exchange between A and B it is necessary to keep in mind that although in A waves propagate in all directions, only the waves with $0 \le \theta < \frac{\pi}{2}$ propagate towards B. Moreover, even a wave with $0 \le \theta < \frac{\pi}{2}$ does not deliver all of its energy to B because part of it is reflected back.

Let $R_H(\omega, \theta)$ be the Fresnel reflection coefficient of a plane wave of frequency ω that has the incidence angle θ. Then, only the $\left(1 - |R_H(\omega, \theta)|^2\right)$-th part of the energy of this wave is transmitted to the domain B. Since the flux across the gap carried by a wave with the incidence angle θ has the value $\mathcal{Q} = c\mathcal{E}\cos\theta$, the total flux carried from A to B by waves of all polarizations, frequencies and incidence angles can be represented as

$$Q_{A\to B}(T_A, Q) = 3c \int_0^{\omega_{\mathrm{D}}}\int_0^{\pi/2} \cos\theta \sin\theta \mathcal{E}(\omega, \theta, T_A, Q) \left(1 - |R_H(\omega, \theta)|^2\right) \mathrm{d}\theta \mathrm{d}\omega. \qquad (17)$$

Then, replacing T_A by T_B and Q by $-Q$ we convert (17) to a similar expression for $Q_{B\to A}(T_B, Q)$, and, finally, taking into account (15), we get the equation

$$Q = \frac{3c}{2} \int_0^{\omega_{\mathrm{D}}}\int_0^{\pi/2} \left(1 - |R_H(\omega, \theta|^2\right) \sin 2\theta \left\{\mathcal{E}(\omega, \theta, T_A, Q) - \mathcal{E}(\omega, \theta, T_B, -Q)\right\} \mathrm{d}\theta \mathrm{d}\omega. \qquad (18)$$

This equation connects the temperatures T_A and T_B with the heat flux Q. If T_A and $T_B \approx T_A$ are fixed, then it can be easily solved for Q numerically, after which the heat transport coefficient can be estimated as (13).

5 Thermal radiation and conduction across nanoscale gaps and numerical results

To estimate heat transport across nanoscale gaps due to phonon tunneling we applied the developed method to the material with the mass-density $\rho = 10400\,\mathrm{kg/m^3}$ and the Debye sound speed $c = 1817$ m/sec corresponding to Silver. The bold solid and dashed lines on Fig. 3 show the heat transfer coefficients computed by the formula (13) for the base temperatures $T_B = 700°\mathrm{K}$ and $T_B = 500°\mathrm{K}$, respectively.

For comparison, Fig. 3 also shows the heat transport coefficients due to thermal radiation, which was computed by the method presented in [1]. The thin solid line marked by circles corresponds to $T_B = 700°\mathrm{K}$ and the thin dashed line marked by stars corresponds to $T_B = 500°\mathrm{K}$. As shown in [1], the heat transport coefficient due to radiation increases as $H \to 0$ at the rate $\sim 1/H^2$, and it approaches a constant asymptote (the Stephan-Boltzman value) as $H \to \infty$. The heat transport coefficient due to phonon tunneling vanishes at the limit $H \to \infty$, but when $H \to 0$ it increases faster than $\sim 1/H^4$, so that at smaller separations phonon conductance dominates radiation, while at larger separations most of heat is carried by radiation. It is seen in Fig. 3 that phonon tunneling exceeds radiation for H less than about 4 nm for the base temperature of 700°K, but this cross-over occurs at about 2 nm for 500°K.

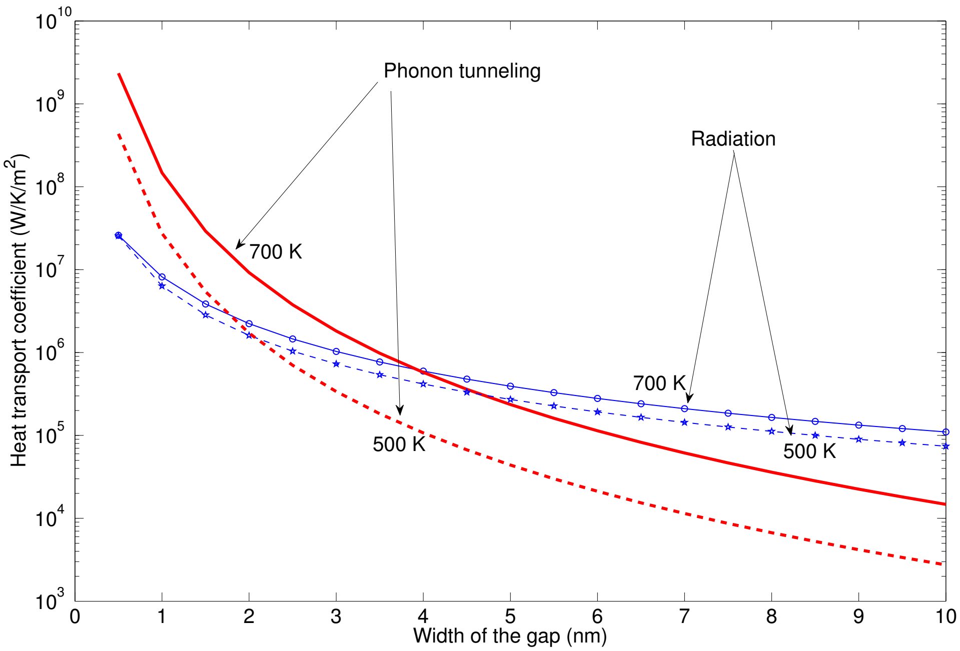


Figure 3: Heat Transport Coefficients

6 Summary and Conclusion

Current HAMR systems are designed for a writing spot of about 20 nm in diameter that must be heated to about 400°C by a Near Field Transducer (NFT), whose tip should be as small as the heated spot, should deliver the power with the density $\sim 10^{11}\,\mathrm{W/m^2}$, should be placed within a couple of nanometers from the heated spot, and should remain sufficiently cool to avoid destruction of the read/write heads, as well as of itself.

It is easy to see that the design of a HAMR system illustrated in Fig. 4 must take into account such processes as: direct heating of the disk and of the NFT by a passing laser energy whose density is sufficient to destroy many materials [12, 13, 14]; heat transport by the air between the NFT and the disk; back-heating of the NFT by phonon tunneling due to van der Waals forces between the molecules in the disk and NFT, and back-heating of NFT by thermal radiation from the hot spot on the disk.

The left figure depicts the initial NFT in which the tip is flat and it delivers heat to a similar area on the disk. The right figure depicts the heat damaged NFT after several hours of use in which the corners have been rounded, the disk heated area is larger, the disk temperature is lower and the NFT to disk spacing is larger. It also depicts the back heating of the NFT from the disk. This damage could be due to the laser supplied energy flux and/or the back heating.

The simultaneous analysis of all of these processes already makes a rather complex problem, but it is further aggravated by the low-nanoscale size of the device, which undermines fundamental assumptions of the conventional theories of radiative heat transport and heat conduction discussed in the Introduction.

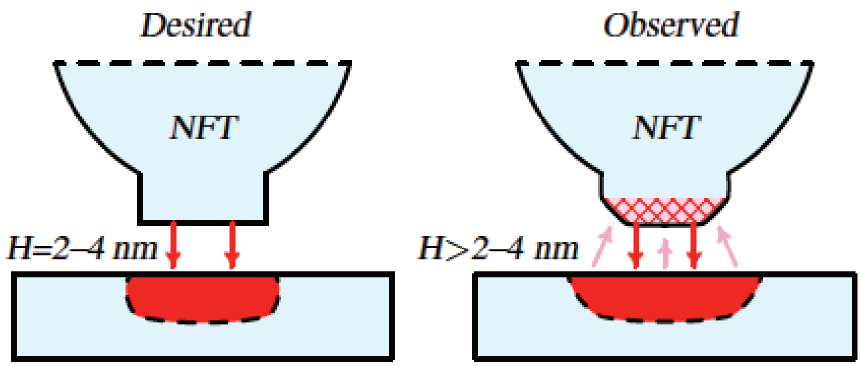


Figure 4: The NFT supplies the energy flux $\sim 10^{11}$W/m^2 to the writing spot. The left figure depicts the initial NFT in which the tip is flat. The right figure depicts the damaged NFT after several hours of use in which the corners have been rounded, the disk heated area is larger, the disk temperature is lower and the NFT to disk spacing is larger. It also depicts the back heating of the NFT from the disk.

Here we considered only heat radiation and heat conduction due to phonon tunneling. Both of these mechanisms of nanoscale heat transport are related with wave propagation processes and can be studied using the extension of Planck's law from equilibrium systems to systems with a non-vanishing heat flux [2]. It was shown that when the gap between the media reduces below a micrometer, the thermal radiation increases as $\sim 1/H^2$ and at $H \sim 4$ nm the radiative heat transport coefficient reaches the level of 10^5 W/m^2. Moreover, as H reduces below $\sim$10 nm, the heat conduction due to phonon tunneling starts increasing at the faster rate $\sim 1/H^p$, with $p \approx 4$, so that this form of heat transport becomes dominating and at $H \sim$2 nm the corresponding heat transport coefficient exceeds 10^7 W/m^2.

Although this study is limited to only two mechanism of heat transport it shows that it may not be possible for the NFT to remain "cool" when the media spot within a few nanometers from it is heated to 400°C because of back heating.

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