Steady state ensembles of thermal radiation in a layered media with a constant heat flux

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Abstract

This paper describes steady-state ensembles of thermally excited electromagnetic radiation in nano-scale layered media with a constant non-vanishing heat flux across the layers. It is shown that Planck's law of thermal radiation, the principle of equivalence, and the laws of wave propagation in layered media, imply that in order for the ensemble of thermally excited electromagnetic fields to exist in a medium consisting of a stack of layers between two half-space, the net heat flux across the layers must exceed a certain threshold that is determined by the temperatures of the half spaces and by the reflective properties of the entire structure. The obtained results provide a way for estimating the radiative heat transfer coefficient of nano-scale layered structures.

1 Introduction

While heat conduction is usually described by the heat equation based on Fourier's Law, this law has been validated only for cases in which the kinetic theory applies. In particular, this theory requires that the dimensions of the regions under consideration significantly exceed the mean free path of heat carrying particles. In nano-scale applications these conditions are often not satisfied. We are concerned here with heat transfer in layered systems in which the thicknesses of the layers may be only a few nanometers, as for example in the proposed heat assisted magnetic recording (HAMR) systems where the layers can be smaller than 5 nm. In these systems the Fourier Law does not apply, and therefore, the heat transfer models used to design them must be re-examined.

It has long been established that heat can be carried by waves [1, 2]. Thus, in dielectric solids, thermal energy is stored in lattice vibrations and heat is carried by waves of lattice vibrations, which are essentially mechanical waves similar to those studied in seismology, acoustics, or structural mechanics, for example. Heat carrying elastic waves are characterized by angular frequencies of the order of $\sim 10^{12}$/s, wave speeds of the order of $\sim 10^3$ m/s, and wavelengths of the order of a few nanometers. When the dimensions of the regions under consideration substantially exceed these wavelengths the waves can form wave packets that can be treated by kinetic theory and hence the heat conduction can be described by the heat equation based on Fourier's Law.

Heat can also be carried by electromagnetic waves radiated by atoms performing thermal motion. Indeed, when the electric charges contained in atoms accelerate they radiate electromagnetic waves that pass part of their energy to other charged particles, which accelerate and heat the medium around them. This mechanism of heat transfer is referred to as radiative transport, and since electromagnetic radiation can propagate long distances even in vacuum, it dominates the heat exchange between bodies separated by considerable distances. The heat carrying electromagnetic waves radiated by a material at room temperature have angular frequencies of the order of $\sim 10^{15}$/s, wave speeds of about $\sim 3 \times 10^8$ m/s, and wavelengths of the order of $\sim 0.5\mu$m $- 5\,\mu$m, which considerably exceeds the dimensions of nanoscale devices but is negligible compared even to many micro-electronic devices.

If the intensity of a wave field has a negligible variance within a distance considerably exceeding the dominant wavelength, then this field can be represented in terms of wave packets that appear as localized particles of a dimension at least one order of magnitude bigger than the dominant wavelength [3, 4]. In such cases the wave packets carry heat in a manner similar to particles of an ideal gas studied in classical kinetic theory, [5]. However, since the kinetic theory is valid only when the dimensions of the particles are considerably smaller than the characteristic dimensions of the domains, the latter should be at least two orders of magnitude larger than the wavelengths of heat carrying wave packets. Therefore, the kinetic theory, Fourier's law, and the heat conduction equation describe heat transport by elastic and electromagnetic waves only when the characteristic dimensions are larger than $\sim 10^{-7}$ m and $\sim 10^{-5}$ m, respectively. Correspondingly, in order to study radiative heat transport in domains smaller than a few microns, or to study heat transport by elastic waves in domains smaller than about a hundred nanometers, it is necessary to take into account specific features of the waves, which are lost in the wave-packet representations of wave fields.

Both elastic and electromagnetic waves are described by the wave equation, which has been studied for more than two centuries. However, most of the studies have been focused on single waves, and little is known about statistical ensembles of heat carrying waves. In this work we study the statistical ensembles of wave fields in a system of two half spaces separated by a stack of layers of nanoscale thickness. For definiteness we consider only ensembles of electromagnetic waves and hence we focus on thermal radiation, but all of the results can be extended to ensembles of elastic waves, and therefore to thermal conduction.

The analysis starts from the description in Section 2 of ensembles of electromagnetic fields in a homogenous space. First we summarize the results of [6] describing ensembles of plane waves in a homogeneous space with a steady heat flux without a temperature differential. Plane waves, however, are not suitable for the analysis of wave fields in layered structures because such waves do not satisfy the interface conditions. So, in order to consider such structures, we also construct ensembles of wave fields with a steady heat flux consisting of pairs of the plane waves, which easily fit any interface conditions in a layered medium .

In Section 3 we study ensembles of electromagnetic waves in a layered structure consisting of two half-spaces $x > H$ and $x < 0$ separated by a stack of layers occupying the region $0 < x < H$. We assume that the half-space $x > H$ has the temperature $T = T_+$ and that the radiation in this domain has a steady heat flux Q parallel to the x-axis. Using the results of Section 2 we model such radiation by a statistical ensemble of the pairs of plane waves in $x > H$. Then, continuing each of these pairs across the layers to the domain $x < 0$, we induce an ensemble of waves in $x < 0$, which also has the same heat flux Q. Therefore, the question arises whether this induced ensemble can be identified with an ensemble of wave fields in $x < 0$ with a given heat flux Q and some temperature T_-. This question is studied in the main part of Section 3 where it is shown that such identification is possible only when the temperature differential $\Delta T = T_- - T_+$ reaches a certain level determined by the heat flux Q. As a result, we get the description of an ensemble of electromagnetic waves in a system of half-spaces separated by layers with a steady heat flux.

The main part of the paper heavily uses the theory of wave propagation in layered media that is well described in acoustics, geophysics, radio science and mathematics literature [7, 8]. However, since this theory is not commonly used in thermal science, its most important results for our needs are summarized in two appendices.

Appendix 1 describes basic electromagnetic fields in a system of two half-spaces $x < 0$ and $x > H$ separated by a stack of layers. In the half-space $x > H$ the basic fields can be represented by a superposition of two plane waves propagating along the directions $\boldsymbol{e} = (e_x, e_y, e_z)$ and $\boldsymbol{e}' = (-e_x, e_y, e_z)$, with $e_x \geq 0$. Similarly, in $x < 0$, the basic fields consist of the pairs of plane waves propagating along $\boldsymbol{d} = (d_x, d_y, d_z)$ and $\boldsymbol{d}' = (-d_x, d_y, d_z)$, with $d_x \geq 0$. In order for the two pairs of plane waves propagating in the different half-spaces to be part of a field in the entire space, the parameters of these pairs must be connected by certain conditions, which are formulated in terms of the reflection coefficient of the stack of layers. Since the reflection coefficient of a stack of layers appears as a critical parameter determining the structure of electromagnetic fields in a layered structure, Appendix 2 describes an efficient recursive algorithm [7], which makes it possible to compute the reflection coefficient of an arbitrary stack of layers. The rest of this appendix discusses several examples illustrating quantitative properties of this coefficient that are important for understanding ensembles of electromagnetic fields in layered structures.

2 Ensembles of electromagnetic fields in a homogeneous space

It is well known that a thermally excited electromagnetic field in an unbounded homogeneous medium can be represented in terms of plane waves with the electric field

$$\boldsymbol{E} = \sqrt{\frac{2}{\epsilon}} P(\omega, T) \mathrm{e}^{\mathrm{i}\omega(e_x x + e_y y + e_z z)/c + \mathrm{i}\beta - \mathrm{i}\omega t} \boldsymbol{n}, \tag{2.1}$$

where $\epsilon = \epsilon(\omega)$ and c are the dielectric function and the wave speed in the medium, β is a random phase shift, $\boldsymbol{e} = (e_x, e_y, e_z)$ is a random unit vector pointing in the direction of propagation, $\boldsymbol{n}$ is a random unit vector (polarization) orthogonal to $\boldsymbol{e}$, and

$$P^2(\omega, T) = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/\kappa T} - 1} \tag{2.2}$$

is the average thermal energy of an oscillator at frequency ω that belongs to an equilibrium ensemble at temperature T.

It is also known that a homogeneous lossless space has an infinite thermal conductance which means that a heat flux may exist in such a space even if its temperature is uniform [9]. In particular [6], in a homogeneous space with a constant heat flux $\boldsymbol{Q} \ll 1$ at the temperature T the average thermal energy density of a plane wave propagating along the unit vector $\boldsymbol{e}$ has the first-order approximation $P^2(\omega Y(\boldsymbol{e}, \boldsymbol{Q}))$, where ω is the frequency and

$$Y(\boldsymbol{e}, \boldsymbol{Q}) = 1 - \frac{\boldsymbol{e} \cdot \boldsymbol{Q}}{c\mathcal{E}(T)}, \tag{2.3}$$

is a Doppler-like factor depending on the total density $\mathcal{E}(T)$ of thermal energy. Correspondingly, the ensemble of thermally excited fields in a space with a small constant heat flux can be represented in terms of plane waves

$$\boldsymbol{E} = \sqrt{\frac{2}{\epsilon}} P(\omega Y(\boldsymbol{e}, \boldsymbol{Q}), T) \mathrm{e}^{\mathrm{i}\omega(e_x x + e_y y + e_z z)/c + \mathrm{i}\beta - \mathrm{i}\omega t}\, \boldsymbol{n}, \tag{2.4}$$

where the random parameters $\boldsymbol{e}$, $\boldsymbol{n}$ and β have the same meanings as in (2.1).

The average energy density $\mathcal{E}(T)$ of the fields (2.4) can be represented as

$$\mathcal{E}(T) = 2 \int_0^\infty P^2(\omega, T) D(\omega) \mathrm{d}\omega, \tag{2.5}$$

where the factor "2" accounts for the two polarizations of the electromagnetic waves, $P^2(\omega, T)$ is defined by (2.2), and $D(\omega)\mathrm{d}\omega$ represents the number of plane waves of one polarization per unit volume with frequencies from the band $(\omega, \omega + \mathrm{d}\omega)$. The density of states $D(\omega)$ of an arbitrary domain may be rather difficult to compute, but for a homogeneous space it has the value $D(\omega) = \omega^2/2\pi^2 c^3$, which reduces (2.5) to the expression $\mathcal{E}(T) = \pi^2 \kappa^4 T^4 / 60 c^3 \hbar^3$, widely known as Planck's law of equilibrium radiation [10, 11, 5].

Plane waves do not form a convenient basis in domains other than a homogeneous medium because the individual plane waves do not satisfy the interface conditions. However, since any wave field can be decomposed into plane waves, the models discussed above can be adjusted to describe ensembles of wave fields in a not-necessarily homogenous medium. In particular, an ensemble of wave fields in a half-space $x < 0$ or $x > H$ can be constructed from the electric fields

$$\boldsymbol{E} = \left(A\boldsymbol{n} \mathrm{e}^{\mathrm{i} e_x x\omega/c} + B\boldsymbol{n}' \mathrm{e}^{-\mathrm{i} e_x x\omega/c} \right) \mathrm{e}^{\mathrm{i}(e_y y + e_z z)\omega/c - \mathrm{i}\omega t}, \qquad e_x \geq 0, \tag{2.6}$$

composed of waves propagating in the directions $\boldsymbol{e} = (e_x, e_y, e_z)$ and $\boldsymbol{e}' = (-e_x, e_y, e_z)$ with matching polarizations $\boldsymbol{n}$ and $\boldsymbol{n}'$, which can be defined by the formula (A1.3).

Consider first an equilibrium ensemble of the waves (2.6). The average energy density of each of these waves can be computed by the thermodynamical expression (2.2) and also by the formula (A1.4) coming from electrodynamics. Therefore, by equating these two representations the coefficients A and B can be represented as

$$A = P(\omega, T) \cos\eta(\omega)\, \mathrm{e}^{\mathrm{i}\beta + \mathrm{i}\alpha}, \qquad B = P(\omega, T) \sin\eta(\omega)\, \mathrm{e}^{\mathrm{i}\beta - \mathrm{i}\alpha}, \tag{2.7}$$

where $\alpha = \alpha(\omega)$ is a relative phase shift, $\beta = \beta(\omega)$ is a common phase shift, which will be suppressed henceforth because it does not play any role, and $\eta = \eta(\omega)$ is a random angle distributed over the interval $0 \leq \eta \leq \frac{\pi}{2}$ in such a way that the average $\langle \cos 2\eta(\omega) \rangle = 0$ vanishes implying that the waves of the ensemble (2.6), (2.7) carry equal amounts of energy in the opposite directions and, thus, the ensemble has no net energy flux. Correspondingly, an equilibrium ensemble of thermally excited electromagnetic wave fields can be represented by a set of random waves

$$\boldsymbol{E} = \sqrt{\frac{2}{\epsilon}} P(\omega, T) \left(\cos\eta \mathrm{e}^{\mathrm{i} e_x x\omega/c + \mathrm{i}\alpha} \boldsymbol{n} + \sin\eta \mathrm{e}^{-\mathrm{i} e_x x\omega/c - \mathrm{i}\alpha} \boldsymbol{n}' \right) \mathrm{e}^{\mathrm{i}(e_y y + e_z z)\omega/c - \mathrm{i}\omega t}, \tag{2.8}$$

with a random relative phase shift α and a random number η whose distribution satisfies

$$\langle \cos 2\eta \rangle = 0. \tag{2.9}$$

It should be mentioned, that although α, β and η may depend on the frequency, we use the abbreviated notation which does not show such dependence wherever it does not cause confusion.

In order to generalize (2.8) to the case when an ensemble of thermally excited wave fields has a constant heat flux $\boldsymbol{Q} = Q\,\widehat{\boldsymbol{x}}$, it suffices to apply a Doppler-like transformation (2.3) to each of the terms in (2.8) and thereby obtain the representations

$$A = \sqrt{\frac{2}{\epsilon}} P(\omega Y(\boldsymbol{e}, Q), T) \cos \eta \mathrm{e}^{\mathrm{i}\alpha} \qquad B = \sqrt{\frac{2}{\epsilon}} P(\omega Y(\boldsymbol{e}', Q), T) \sin \eta \mathrm{e}^{-\mathrm{i}\alpha}, \tag{2.10}$$

which can be conveniently re-written as

$$A = \sqrt{\frac{2}{\epsilon}} P(\omega, T) a(Q, 0; \omega) \cos \eta \,\mathrm{e}^{\mathrm{i}\alpha}, \qquad B = \sqrt{\frac{2}{\epsilon}} P(\omega, T) a(Q, 0; \omega) \sin \eta \,\mathrm{e}^{-\mathrm{i}\alpha}, \tag{2.11}$$

where the coefficients

$$\begin{aligned} a(Q, \Delta T; \omega) &= \frac{P(\omega Y(\boldsymbol{e}, Q), T + \Delta T)}{P(\omega, T)} \cdot \frac{\cos (\eta Y(\boldsymbol{e}, Q))}{\cos \eta}, \\ b(Q, \Delta T; \omega) &= \frac{P(\omega Y(\boldsymbol{e}', Q), T + \Delta T)}{P(\omega, T)} \cdot \frac{\sin (\eta Y(\boldsymbol{e}', Q))}{\sin \eta}, \end{aligned} \tag{2.12}$$

reduce to $a = b = 1$ when $Q = \Delta T = 0$, but may significantly deviate from unity even when Q and ΔT are quite small.

It is important to note that $a(Q, \Delta T; \omega)$ and $b(Q, \Delta T; \omega)$ depend not only on the listed parameters but also on the temperature T, the direction of the heat flux $\boldsymbol{Q}$, the direction $\boldsymbol{e} = (e_x, e_y, e_z)$ of the waves from (2.6), and on the wave speed c in the medium. None of these parameters can be neglected, but for transparency we will suppress them whenever doing so does not cause confusion. Finally, if $Q \neq 0$ then $a(Q, \Delta T; \omega)$ and $b(Q, \Delta T; \omega)$ also depend on the parameter $\eta = \eta(\omega)$ which is involved through the last factors in (2.12). As for the relative phase shift $\alpha = \alpha(\omega)$, although it is also affected by the Doppler transform, it may be left unmodified, because its transformation may always be compensated by the absolute phase shift $\beta = \beta(\omega)$, which does not show up in any important formulas and, thus, may be be assigned any convenient value.

3 Ensembles of electromagnetic fields in a layered structure

The previous section implies that if the half-spaces $x < 0$ and $x > H$ have the temperatures T_- and T_+, respectively, then the ensembles of waves with a small heat flux $\boldsymbol{Q}$ can be represented in these domains by random sets of waves (2.6) with the amplitudes (A_-, B_-) and (A_+, B_+) defined independently of each other by the formulas (2.10). On the other hand, the theory of wave propagation in layered media outlined in Appendix 1 implies that the pairs (A_-, B_-) and (A_+, B_+) are not independent but are connected by the equations (A1.20). Therefore, since $A_\pm$ and $B_\pm$ depend on the heat flux $\boldsymbol{Q}$ and the temperatures T_+, T_-, these three quantities appear to be connected by the two equations, so that one of them may be represented in terms of the other two.

In order to connect the statistical and thermodynamical characteristics of the ensembles of waves in $x < 0$ and $x > H$ we assume that these domains have the temperatures

$$T_- = T + \Delta T, \qquad T_+ = T, \tag{3.1}$$

respectively, and that the heat flux in both half-spaces is directed along the x-axis, so that

$$\boldsymbol{Q}_- = \boldsymbol{Q}_+ = Q\,\widehat{\boldsymbol{x}}. \tag{3.2}$$

Then, as shown in Section 2, the wave ensemble in the domain $x < 0$ can be represented as a set of waves (2.6) with the amplitudes

$$\begin{aligned} A_- &= \sqrt{\frac{2}{\epsilon_-}} P(\omega, T) a_-(Q, \Delta T; \omega) \cos\eta_- \mathrm{e}^{\mathrm{i}\alpha_-}, \\ B_- &= \sqrt{\frac{2}{\epsilon_-}} P(\omega, T) b_-(Q, \Delta T; \omega) \sin\eta_- \mathrm{e}^{-\mathrm{i}\alpha_-}, \end{aligned} \tag{3.3}$$

and the ensemble in $x > H$ can be represented as a set of waves (2.6) with the amplitudes

$$\begin{aligned} A_+ &= \sqrt{\frac{2}{\epsilon_+}} P(\omega, T) a_+(Q, 0; \omega) \cos\eta_+(\omega) \mathrm{e}^{\mathrm{i}\alpha_+}, \\ B_+ &= \sqrt{\frac{2}{\epsilon_+}} P(\omega, T) b_+(Q, 0; \omega) \sin\eta_+(\omega) \mathrm{e}^{-\mathrm{i}\alpha_+}, \end{aligned} \tag{3.4}$$

where $\alpha_\pm = \alpha_\pm(\omega)$ are random phase shifts, $\eta_\pm = \eta_\pm(\omega)$ are random angles distributed in such a way that $\langle \cos 2\eta_\pm(\omega) \rangle = 0$, and the pairs of coefficients a_+, b_+ and a_-, b_- are defined by the formulas (2.12) applied to the media in $x > H$ and $x < 0$, respectively. As follows from the results of Appendix 1, if the fields (2.6) with the coefficients (3.4) and (3.3) continue across the half-spaces to the entire structure, then the coefficients (3.4) and (3.3) are connected by the equations (A1.20), which reduce, in a view of (A1.17), to the form

$$\begin{aligned} &\gamma \left(a_-^2 \cos^2\eta_- + b_-^2 \sin^2\eta_- \right) (1 - R^2) \\ &\qquad = (1 + R^2) \left(a_+^2 \cos^2\eta_+ + b_+^2 \sin^2\eta_+ \right) + 2R a_+ b_+ \sin 2\eta_+ \cos 2\alpha_+, \end{aligned} \tag{3.5}$$

$$\gamma \left(a_-^2 \cos^2\eta_- - b_-^2 \sin^2\eta_- \right) = a_+^2 \cos^2\eta_+ - b_+^2 \sin^2\eta_+, \tag{3.6}$$

where $\eta_\pm$ and α_+ are three unknowns, and

$$\gamma = \frac{\nu\sqrt{1 - \nu^2(1 - \cos^2\theta_+)}}{\cos\theta_+}, \qquad \nu = \frac{c_-}{c_+}, \tag{3.7}$$

is a coefficient determined by the incidence angle θ_+ in the half-space $x > H$ and by the wave speeds c_+, c_- in the half-spaces $x > H$, $x < 0$, respectively.

Since two equations for three unknowns typically have infinitely many solutions, there are, in general, infinitely many triplets $(\eta_+, \eta_-, \alpha_+)$ which satisfy (3.5), (3.6), and thus there is an infinite set of electromagnetic fields compatible with given thermodynamical properties of the considered layered structure. Then, assuming that the admissible triplets are random variables satisfying conditions $\langle \cos 2\eta_\pm \rangle = 0$ we get a statistical ensemble of electromagnetic fields with the given heat flux and temperatures of the half-spaces. The above implies that in order to describe such an ensemble it suffices to formulate the conditions on the temperatures $T_\pm$ and the flux Q, which guarantee that the equations (3.5), (3.6) have solutions, and then find these solutions.

To analyze equations (3.5), (3.6) we represent $\cos^2 \eta_\pm$ and $\sin^2 \eta_\pm$ in terms of $\cos 2\eta_\pm$ and get equivalent equations

$$\gamma(1 - R^2)(p_- + s_- \cos 2\eta_-) = (1 + R^2)(p_+ + s_+ \cos 2\eta_+) + 2Ra_+b_+ \sin 2\eta_+ \cos 2\alpha_+, \tag{3.8}$$
$$\gamma(s_- + p_- \cos 2\eta_-) = s_+ + p_+ \cos 2\eta_+, \tag{3.9}$$

where

$$p_\pm = \frac{1}{2}\left(a_\pm^2 + b_\pm^2\right), \qquad s_\pm = \frac{1}{2}\left(a_\pm^2 - b_\pm^2\right). \tag{3.10}$$

Then, eliminating $\cos 2\eta_-$ we obtain an equation

$$L \cos 2\alpha_+ \sin 2\eta_+ + M \cos 2\eta_+ + N = 0, \tag{3.11}$$

with the coefficients

$$L = 2p_-a_+b_+R, \tag{3.12}$$
$$M = p_-s_+(1 + R^2) - s_-p_+(1 - R^2), \tag{3.13}$$
$$N = p_+p_-(1 + R^2) - \left(s_+s_- + \gamma p_-^2 - \gamma s_-^2\right)(1 - R^2). \tag{3.14}$$

It is worth mentioning that although the equation (3.11) does not involve η_- explicitly, in cases with $Q \neq 0$ it involves this variable implicitly. Indeed, formulas (3.11) depend on p_- and s_-, which are defined by (3.10) in terms of a_- and b_-, which, as mentioned in the end of Section 2, depend on η_- through the last factors of the (2.12). Therefore, in general, (3.11) cannot be considered alone, but must be coupled with either of the equations (3.8) or (3.9). As a result we get a rather complicated non-linear system of equations (3.9), (3.11), which can only be considered numerically. However, to get a rough quantitative idea about the character of the solutions it suffices to analyze (3.11) assuming that the coefficients (3.14) do not depend on η_-.

Assuming that the equation (3.11) does not involve η_-, we apply some trigonometric transformations and reduce (3.11) to a quadric equation

$$(N - M) \tan^2 \eta_+ + 2L \cos 2\alpha_+ \tan \eta_+ + (N + M) = 0, \tag{3.15}$$

which can be satisfied by a pair (η_+, α_+) if and only if

$$L^2 - (N - M)(N + M) = L^2 + M^2 - N^2 \geq 0. \tag{3.16}$$

Indeed, (3.16) guarantees the existence of α_+ satisfying $\cos^2 \alpha_+ \geq (N^2 - M^2)/L^2$, and for each of such α_+, the quadric equation (3.15) for $\tan \eta_+$ has real-valued solutions.

Consider first the equation (3.11) in the equilibrium case when $Q = \Delta T = 0$. In this case

$$a_\pm = b_\pm = p_\pm = 1, \quad s_\pm = 0, \qquad L = 2R, \quad M = 0, \quad N = 1 + R^2 - \gamma(1 - R^2), \tag{3.17}$$

so that the equation (3.11) and the condition of solvability (3.16) reduce to

$$\sin 2\eta_+ \cos 2\alpha_+ = -\widetilde{R}, \qquad |\cos 2\alpha_+| > \widetilde{R}, \tag{3.18}$$

with

$$\widetilde{R} = \frac{1 + R^2 - \gamma(1 - R^2)}{2R} \,. \tag{3.19}$$

Obviously, these equations are solvable only when $|\widetilde{R}| \le 1$. In particular, if $\widetilde{R} = 1$ or $\widetilde{R} = -1$, then $\alpha_+ = 0$ or $\alpha_+ = \frac{\pi}{2}$, respectively, and $\eta_+ = \frac{\pi}{4}$. In a more general case $|\widetilde{R}| < 1$, there are infinitely many solutions determined by the formulas

$$\arcsin(\widetilde{R}) \le 2\eta_+ \le \pi - \arcsin(\widetilde{R}), \qquad 2\alpha_+ = \arccos\left(\frac{-\widetilde{R}}{\sin 2\eta_+}\right). \tag{3.20}$$

This expression essentially describes an equilibrium ensemble of thermally excited electromagnetic waves at any temperature $T = T_- = T_+$ of the medium. It is worth mentioning that if $\gamma = 1$ which corresponds to the case when the half-spaces $x > H$ and $x < 0$ are made from the same material, the condition of solvability (3.16) is automatically satisfied because in this case $\widetilde{R} = R \le 1$.

Next we consider a hypothetical non-equilibrium case with $Q = 0$ and $\Delta T \neq 0$. In this case

$$a_+ = b_+ = p_+ = 1, \qquad s_+ = 0, \tag{3.21}$$

$$a_- = b_- = p_- = \frac{P(\omega, T + \Delta T)}{P(\omega, T)} = \sqrt{\frac{\mathrm{e}^{\hbar\omega/kT} - 1}{\mathrm{e}^{\hbar\omega/(\kappa T + \kappa\Delta T)} - 1}}\,, \qquad s_- = 0, \tag{3.22}$$

and the general equation (3.11) again reduces to the form (3.18), but this time $\widetilde{R}$ is defined as

$$\widetilde{R} = \frac{1 + R^2 - \gamma p_-(1 - R^2)}{2R}\,, \tag{3.23}$$

which ensures that (3.18), (3.23) has no solutions at high frequencies ω. Indeed, if $\Delta T < 0$, then $p_-(\omega) \to \infty$ as $\omega \to \infty$, and (3.18), (3.23) cannot be satisfied because its right-hand side increases to infinity at high frequencies, but the left-hand side is bounded for all ω. If $\Delta T > 0$, then at high frequencies $p_-(\omega) \to 0$, and $\widetilde{R} \to (1 + R^2)/2R > 1$, which also makes it impossible to satisfy (3.18).

In the general case $\Delta T \neq 0$ and $Q \neq 0$, the main equation (3.11) does not have any solutions for some combinations of Q and ΔT, but it may have some solutions for other combinations. Indeed, if $\Delta T \neq 0$, then, as shown above, this equation does not have solutions if $Q = 0$. On the other hand, if $Q \gg 1$, then the definitions of $a_\pm$ and $b_\pm$ imply the estimates

$$a_\pm \gg b_\pm, \qquad p_\pm \sim s_\pm \sim a_\pm^2, \qquad L \sim 2a_-^2 a_+ b_+ R, \qquad M \sim N \sim a_-^2 a_+^2 R^2, \tag{3.24}$$

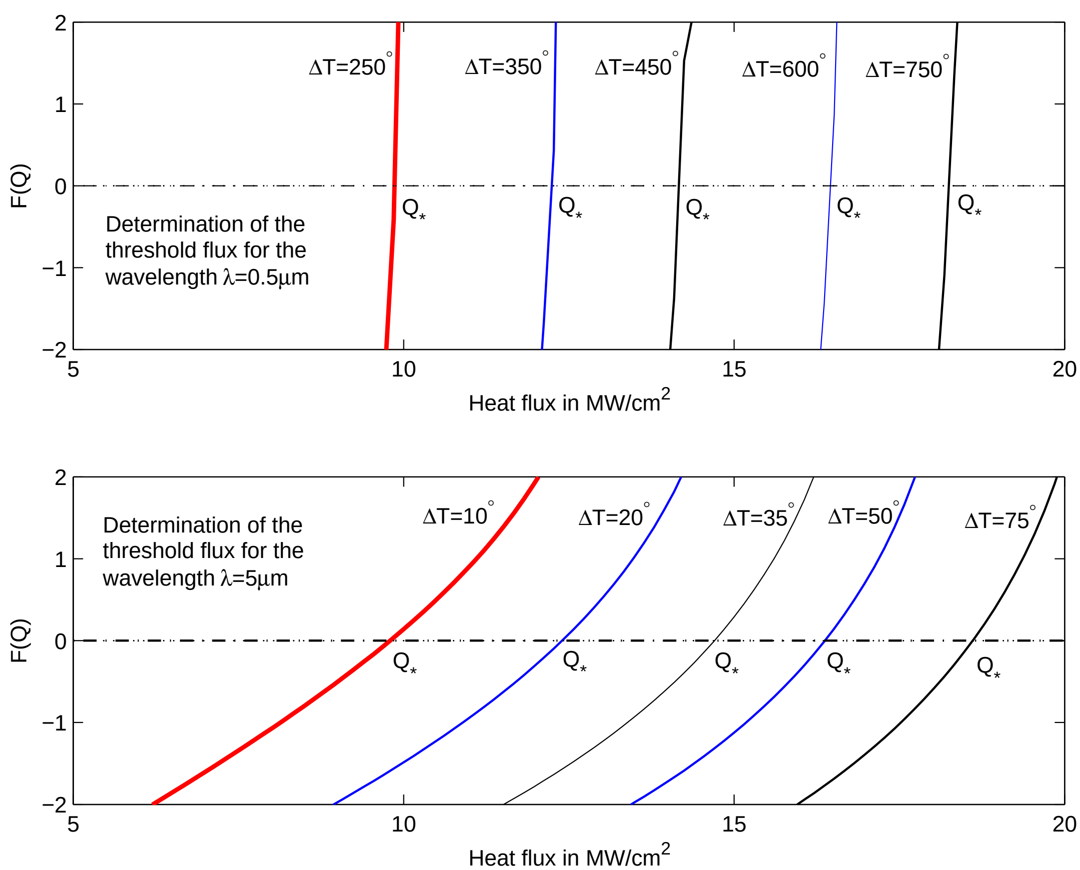
which ensure the inequality (3.16) that guarantees the existence of the solutions of (3.15). This implies that for every temperature differential $\Delta T \neq 0$, there is a minimal heat flux Q for which the equation (3.15) has solutions determining an ensemble of wave fields in the system of half spaces separated by a stack of layers.

The above analysis is illustrated in Fig. 1, which plots the function

$$F(Q; R) = \ln\left[L^2(Q) + M^2(Q)\right]) - \ln\left[N^2(Q)\right] \tag{3.25}$$

for different temperature differentials ΔT and wavelengths of the radiation. We consider a five-layer structure similar to that appearing in HAMR systems. These structures include a glass half-space ($\epsilon = 5$) covered by a 20 nm ferromagnetic layer ($\epsilon = 2$) with a 2 nm carbon overcoat ($\epsilon = 2$) and separated by an air gap from another glass half-space ($\epsilon = 5$), which models a transducer.

The top and the bottom plots correspond to the wavelengths $\lambda = 0.5\,\mu$m and $\lambda = 5\,\mu$m located on the short-wave margin and in the middle of the spectrum of thermal radiation at temperature $300\,^\circ$K, respectively. Each of these plots has five individual curves corresponding to five different temperature differentials ΔT, selected as $\Delta T = 250, 350, 450, 600, 750\,^\circ$K on the top plot, and as $\Delta T = 10, 20, 35, 50, 75\,^\circ$K, on the bottom plot. Each of the curves intersects the axis $F = 0$ at some point $Q = Q_*$, and since the inequality $F(Q) \geq 0$ is equivalent the condition (3.16), this implies that the non-equilibrium ensemble of wave fields with corresponding temperature differential and frequency exists for any $Q \geq Q_*$.



For any given base temperature T_+, temperature differential ΔT and wavelength λ, ensembles of thermally excited electromagnetic fields with the heat flux Q exist if $Q \geq Q_*$, where Q_* is the equation $F(Q_*) = 0$. The longer waves have higher threshold Q_* because such waves are less affected by the stack of layers of given thickness.

Figure 1: Illustration of the condition of solvability

These graphs confirm that for a fixed temperature differential the critical heat flux Q_* increases with the increase of the wavelength because longer waves should be less affected by a structure of fixed width. These graphs also confirm that the higher temperature differential corresponds to the higher value of the critical heat flux. Finally, these graphs make it possible to estimate the temperature differential needed to supply a certain heat flux to a half space with a pre-defined temperature. Assume, for example, that the half-space $x > H$ at the base temperature $T_+ = 700\,^\circ\mathrm{K}$ needs to the get a heat flux of the order $Q \sim 10\,\mathrm{MW/cm^2}$. Then, the leftmost graphs from Fig. 1 suggest that if all the radiation has the dominant frequency then this heat flux is provided by the temperature differential $\Delta T \sim 10\,^\circ\mathrm{K}$, and if all the radiation has the frequency at the short-wave margin, than $\Delta T \sim 250\,^\circ\mathrm{K}$. Therefore, comparing these results we expect that for thermal radiation with Planck's spectrum, the heat flux $Q \sim 10^{11}\,\mathrm{W/m^2}$ may be provided by $\Delta T \sim 100\,^\circ\mathrm{K}$, and finally, since Fig. 1 considers only energy transfer by waves of normal incidence, which are the most efficient carriers in the direction across the layers, we may expect that the natural radiation in arbitrary directions would need to be of the order $\Delta T \sim 300\,^\circ\mathrm{K}$.

4 Conclusion

The above analysis describes steady-state ensembles of thermally excited electromagnetic fields for layered structures with a constant heat flux across the layers. This extends the results of [6] where Planck's law of equilibrium thermal radiation is generalized to a uniform space with a constant heat flux, and it also extends the description of equilibrium ensembles of radiation in layered structures obtained in [12]. This is accomplished by combining the results from [6, 12] with the theory of wave propagation in layered media [7].

The obtained results for the steady-state ensembles of electromagnetic waves in layered structures establish connections between the temperatures of the boundary half-spaces, and this opens a way for the estimations of the radiative thermal transport across such structures. In the examples we considered only electromagnetic waves of one polarization propagating perpendicular to the interfaces, but the methods of the analysis of these examples can be straightforwardly extended to plane waves of any polarization propagating in any direction.

The approach to the analysis of steady-state ensembles of waves in layered structures was introduced in [13, 14, 15], where it was applied to the phenomenon of interface thermal resistance widely known as Kapitsa resistance, which is related with acoustic waves in a similar way as radiative heat transport is related with electromagnetic waves. It was shown in [13, 14, 15] that the interface thermal resistance could be analyzed by a rather general procedure consisting of three distinctive steps: construct ensembles of eigenfields with constant heat flux in homogeneous media at uniform temperatures, construct an equilibrium ensemble of eigenfields in a layered structure, then combine the results of the first two steps and construct an ensemble of eigenfields in the layered structure with a steady heat flux. In [13, 14, 15] these steps were proposed and tested in the simplest layered structure consisting of two adjacent half-spaces. In order to extend these promising first results, we first extended Planck's law of equilibrium radiation to radiation with a constant heat flux [6]. Then we described equilibrium ensembles of thermal radiation in an arbitrary layered structure [12], and here we complete the cycle by describing steady-state thermal radiation with a constant heat flux in arbitrary layered media. These results build the foundation for understanding radiative heat transfer in nanoscale layered structures, and they will be useful for the analysis and design of practical devices.

Acknowledgment

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Appendix 1: Electromagnetic waves in two half-spaces connected by a stack of layers

Any electromagnetic wave in a homogeneous space occupied by a medium with the dielectric and magnetic constants ϵ and μ can be represented as a linear combination of plane waves characterized by the electric and magnetic fields

$$\begin{aligned} \boldsymbol{E} &= E\mathrm{e}^{\mathrm{i}\omega(e_x x+e_y y+e_z z)/c-\mathrm{i}\omega t}\,\boldsymbol{n}, \\ \boldsymbol{B} &= \frac{E}{c}\,\mathrm{e}^{\mathrm{i}\omega(e_x x+e_y y+e_z z)/c-\mathrm{i}\omega t}\,\boldsymbol{e}\times\boldsymbol{n} = \frac{\mathrm{i}}{\omega}\boldsymbol{\nabla}\times\boldsymbol{E}, \end{aligned} \tag{A1.1}$$

where ω is the frequency, $\boldsymbol{e} = (e_x, e_y, e_z)$ is the unit vector of the direction of propagation,

$$c = \frac{1}{\sqrt{\epsilon\mu}} \tag{A1.2}$$

is the wave speed, and $\boldsymbol{n}$ is a unit vector of polarization. The polarization $\boldsymbol{n}$ may take any direction orthogonal to the direction of propagation $\boldsymbol{e}$, which implies that it can be represented as a linear combination of two mutually orthogonal vectors

$$\boldsymbol{n}_\perp \equiv \boldsymbol{e}\times\,\widehat{\boldsymbol{x}}, \qquad \text{or} \qquad \boldsymbol{n}_\parallel \equiv \boldsymbol{e}\times\boldsymbol{n}_\perp\,, \tag{A1.3}$$

the first of which is parallel to the plane $x = 0$, while both $\boldsymbol{n}_\perp$ and $\boldsymbol{n}_\parallel$ are orthogonal to the direction of propagation $\boldsymbol{e}$. Waves (A1.1) with $\boldsymbol{n} = \boldsymbol{n}_\perp$ and $\boldsymbol{n} = \boldsymbol{n}_\parallel$ are referred to as waves with perpendicular and parallel polarizations with respect to the plane $x = 0$, respectively.

Any plane wave (A1.1), independently of its polarization, carries the average energy density (energy per unit volume per cycle of oscillations)

$$\mathcal{E} = \frac{1}{4}\left(\epsilon|\boldsymbol{E}|^2 + \frac{1}{\mu}|\boldsymbol{B}|^2\right) = \frac{1}{2}\,\epsilon E^2, \tag{A1.4}$$

and the average energy flux (energy per unit time per unit area per cycle of oscillations)

$$\boldsymbol{S} = S\boldsymbol{e}, \qquad S = \frac{1}{\mu}|\boldsymbol{E}\times\boldsymbol{B}| = c\mathcal{E}, \tag{A1.5}$$

which flows along the direction of wave propagation $\boldsymbol{e}$.

We now consider a structure consisting of two half-spaces $x < 0$ and $x > H$ separated by a stack of layers $0 < x_1 < \cdots < x_j < H$ whose properties are not relevant for our purposes.

Let ϵ_- and ϵ_+ denote the dielectric constants in $x < 0$ and $x > H$, respectively. Similarly, let c_- and c_+ be the wave speeds in these half-spaces. Then, any electromagnetic wave in these domains can be represented as a superposition of piece-wise plane waves with the electric components

$$\boldsymbol{E}(x, y, z, t; \boldsymbol{e}, \boldsymbol{d}) = \begin{cases} \boldsymbol{E}_+(x, y, z; \boldsymbol{e})\mathrm{e}^{-\mathrm{i}\omega t}, & x > H, \\ \boldsymbol{E}_-(x, y, z; \boldsymbol{d})\mathrm{e}^{-\mathrm{i}\omega t}, & x < 0, \end{cases} \tag{A1.6}$$

where the directions $\boldsymbol{e} = (e_x, e_y, e_z)$ and $\boldsymbol{d} = (d_x, d_y, d_z)$ are connected by the Snell's formulas

$$d_z = \nu e_z, \quad d_y = \nu e_y, \quad d_x = \sqrt{1-\nu^2(e_x^2+e_x^2)}, \qquad \nu = \frac{c_-}{c_+}\,, \tag{A1.7}$$

and

$$
\begin{aligned}
\boldsymbol{E}_+(x,y,z;\boldsymbol{e}) &= \left(A_+\boldsymbol{n}_+\mathrm{e}^{\mathrm{i}e_x x\omega/c_+} + B_+\boldsymbol{n}'_+\mathrm{e}^{-\mathrm{i}e_x x\omega/c_+}\right)\mathrm{e}^{\mathrm{i}(e_y y+e_z z)\omega/c_+},\\
\boldsymbol{E}_-(x,y,z;\boldsymbol{d}) &= \left(A_-\boldsymbol{n}_-\mathrm{e}^{\mathrm{i}d_x x\omega/c_-} + B_-\boldsymbol{n}'_-\mathrm{e}^{-\mathrm{i}d_x x\omega/c_-}\right)\mathrm{e}^{\mathrm{i}(d_y y+d_z z)\omega/c_-},
\end{aligned}
\tag{A1.8}
$$

where (A_-,B_-) and (A_+,B_+) are complex-valued coefficients connected by the linear equations

$$
\begin{aligned}
A_- &= T_{11}A_+ + T_{12}B_+\,,\\
B_- &= T_{21}A_+ + T_{22}B_+\,,
\end{aligned}
\tag{A1.9}
$$

whose coefficients T_{mn} have a clear physical meaning as described below. As for the polarization vectors, $\boldsymbol{n}_\pm$ and $\boldsymbol{n}'_\pm$, one of them may take arbitrary values and then the others become uniquely defined. For definiteness, assume that $\boldsymbol{n}_+$ is independent. Then, $\boldsymbol{n}_+$ takes any of two values defined by (A1.3), and $\boldsymbol{n}'_+$ is defined by the same formula but with $\boldsymbol{e}$ replaced by $\boldsymbol{e}' = (-e_x, e_y, e_z)$. Similarly, $\boldsymbol{n}_-$ and $\boldsymbol{n}'_-$ are defined by the formula (A1.3) with $\boldsymbol{e}$ and $\boldsymbol{e}'$ replaced by $\boldsymbol{d}$ and $\boldsymbol{d}' = (-d_x, d_y, d_z)$, respectively. Since any of the vectors $\boldsymbol{e}'$, $\boldsymbol{d}$ and $\boldsymbol{d}'$ is completely defined by $\boldsymbol{e}$, the polarization vector $\boldsymbol{n}_+$ completely determines all polarization vectors involved in (A1.8).

To identify the coefficients T_{mn} it suffices to consider the fields

$$
\begin{aligned}
\boldsymbol{E}_- &= \boldsymbol{n}_+\mathrm{e}^{\mathrm{i}\omega(e_x x+e_y y+e_z z)/c_-} + B_-\boldsymbol{n}'_+\mathrm{e}^{\mathrm{i}\omega(-e_x x+e_y y+e_z z)/c_-},\\
\boldsymbol{E}_+ &= A_+\boldsymbol{n}_-\mathrm{e}^{\mathrm{i}\omega(d_x x+d_y y+d_z z)/c_+}\,,
\end{aligned}
\tag{A1.10}
$$

which appear as a particular case of (A1.8) with $A_- = 1$ and $B_+ = 0$. Then, treating the first component of $\boldsymbol{E}_-$ as an incident wave arriving at the interface $x = 0$ from the domain $x < 0$, we identify the second component of $\boldsymbol{E}_-$ as the wave reflected back to $x < 0$, and $\boldsymbol{E}_+$ as the wave transmitted to $x > H$. Correspondingly, $B_- = R_-$ and $A_+ = K_-$ are identified as the reflection and transmission coefficients R_- and K_- of the plane wave $\mathrm{e}^{\mathrm{i}\omega(e_x x+e_y y+e_z z)/c_-}$ arriving from $x < 0$. Similarly, $A_+ = K_-$ can be identified with the transmission coefficient of this plane wave from the domain $x < 0$ to the domain $x > H$. On the other hand, applying (A1.9) to the considered case with $A_- = 1$ and $B_+ = 0$ we get the equations

$$
1 = T_{11}A_+, \qquad B_- = T_{12}A_+,
\tag{A1.11}
$$

and, combining them using the above reasoning, we arrive at the representations

$$
T_{11} = \frac{1}{K_-}, \qquad T_{12} = \frac{R_-}{K_-}.
\tag{A1.12}
$$

Then, complex-conjugating (A1.10) we readily find that

$$
T_{22} = T_{11}^* = \frac{1}{K_-^*}, \qquad T_{21} = T_{12}^* = \frac{R_-^*}{K_-^*},
\tag{A1.13}
$$

where the asterisk denotes the operation of complex conjugation.

Since the reflection and transmission coefficients may be complex-valued, it is convenient to represent them as

$$
K_- = |K_-|\mathrm{e}^{\mathrm{i}\phi_K}, \qquad R_- = R\mathrm{e}^{\mathrm{i}\phi_R}, \qquad R = |R_-|,
\tag{A1.14}
$$

and then re-arrange (A1.9) with the aid of (A1.12) and (A1.13), to the equations

$$
\begin{aligned}
|K_-|A_- &= \mathrm{e}^{-\mathrm{i}\phi_K} A_+ + R\mathrm{e}^{\mathrm{i}(\phi_K-\phi_R)} B_+ \,, \\
|K_-|B_- &= \mathrm{e}^{\mathrm{i}\phi_K} B_+ + R\mathrm{e}^{-\mathrm{i}(\phi_K-\phi_R)} A_+ \,,
\end{aligned}
\tag{A1.15}
$$

which insure that the waves (A1.8) are parts of an electromagnetic field defined in the system of two half-spaces $x < 0$ and $x > H$ separated by a stack of layers.

The reflection and transmission coefficients R_- and K_- depend on the polarization and direction of the plane waves, as well as on the parameters of all layers. These coefficients can be computed by standard methods of the theory of wave propagation in layered media [7], but for our purposes it suffices to know only a few relationships that follow from the law of energy conservation. First, we observe that independently of the structure of the layers in the domain $0 < x < H$, the reflection and transmission coefficients obey the inequality

$$
\epsilon_- c_- d_x \geq \epsilon_- c_- d_x R^2 + \epsilon_+ c_+ e_x |K_-|^2. \tag{A1.16}
$$

Indeed, the left-hand side of (A1.16) represents the energy arriving per unit of time at an unit area of the interface $x = 0$ from $x < 0$; the first term of the right-hand side equals to the energy leaving an unit area of $x = 0$ towards $x < 0$, and the second term of the right-hand side represents the energy leaving an unit area of the interface $x = H$ toward the domain $x > H$. Therefore, the inequality (A1.16) immediately follows from by the law of energy conservation.

It is worth mentioning that the strict inequality in (A1.16) may occur even if all layers are lossless. For example, for certain directions $\boldsymbol{e}$, the field (A1.6) may excite surface waves propagating parallel to the plane $x = 0$. Such waves take away some of the arriving energy, resulting in a strict inequality in (A1.16). However, in this study we limit ourselves to the cases when the reflection and transmission coefficients are connected by the identity

$$
\epsilon_+ |K_-|^2 = \epsilon_- \gamma \left(1 - R^2\right), \tag{A1.17}
$$

where

$$
\gamma = \frac{c_- d_x}{c_+ e_x} = \frac{\nu\sqrt{1-\nu^2(1-e_x^2)}}{e_x}, \tag{A1.18}
$$

which follows from (A1.16) under the assumption that no energy stays in the layers.

Despite its general form, the last relationship provides valuable information about the reflection and transmission coefficients. Consider, for example, the field (A1.6) in the case of full reflection. Assume, for definiteness that $c_+ > c_-$, so that $c_+/c_- > 1$. Then, the Snell law (A1.7) implies that if the direction $\boldsymbol{d} = (d_x, d_y, d_z)$ of propagation in the domain $x < 0$ satisfies the condition

$$
d_x^2 < 1 - \nu^2, \tag{A1.19}
$$

then the direction of propagation $\boldsymbol{e} = (e_x, e_y, e_z)$ in $x > H$ has an imaginary component $e_x = \mathrm{i}|e_x|$. In this case, the field $\boldsymbol{E}_+$ from (A1.8) exponentially decays as $x \to \infty$, which implies that there is no propagation along the x-axis, and that, therefore, the transmission coefficient vanishes, i.e. $K_- = 0$. Then, (A1.17) implies that there is full reflection, i.e., $R = 1$. This example shows that if the half-spaces $x < 0$ and $x > H$ have different wave speeds, then only part of the fields (A1.6) participate in the energy transfer between the half-spaces. In particular, if $c_+ > c_-$, then the fields (A1.6) with d_x satisfying (A1.19) do not participate in energy exchange between the half spaces.

It should finally be mentioned that the equations (A1.15) imply simple relationships

$$\frac{\gamma\epsilon_-}{\epsilon_+}(1-R^2)\left(|A_-|^2+|B_-|^2\right)=(1+R^2)(|A_+|^2+|B_+|^2)+4\,\mathrm{Re}\left(A_+B_+^*R\mathrm{e}^{\mathrm{i}(\phi_R-2\phi_K)}\right), \quad \text{(A1.20)}$$

$$\frac{\gamma\epsilon_-}{\epsilon_+}(1-R^2)\left(|A_-|^2-|B_-|^2\right)=(1-R^2)(|A_+|^2-|B_+|^2), \quad \text{(A1.21)}$$

which guarantee that the field $\boldsymbol{E}_+$ from (A1.8) with given A_+ and B_+ can be continued to a field $\boldsymbol{E}_-$ from (A1.8) with some A_- and B_- satisfying (A1.20), (A1.21). Indeed, if the right-hand sides of the last formulas are known then $|A_-|$ and $|B_-|$ are uniquely defined, and there exist phase shifts α_- and β_- for which $A_-=|A_-|\mathrm{e}^{\mathrm{i}\beta_-+\mathrm{i}\alpha_-}$ and $B_-=|B_-|\mathrm{e}^{\mathrm{i}\beta_--\mathrm{i}\alpha_-}$.

Appendix 2: The reflection coefficient of the layered structure

The above results show that the structure of the ensemble of thermally excited electromagnetic fields in a system of two half-spaces separated by a stack of nanoscale layers is essentially determined by the reflection coefficients of plane incident waves arriving from one of the half-spaces with all possible incidence angles . This implies that in order to study thermal conduction/resistance of a nanoscale layered structure it is necessary to have an efficient method for the computations of the reflection coefficients of such structures. For our purposes, it suffices to use a simple recursive method discussed in detail in [7] and briefly summarized below.

Let the space be divided by the planes $x=a_1=0$, $x=a_2$, $\ldots$, $x=a_{J-1}=H$ into J domains, including two half-spaces $x<0$, $x>H$ and $(J-2)$ layers, as shown in Fig. 2. Let these domains be enumerated by integers from $j=1$ to $j=J$, with the numbers $j=1$ and $j=J$ assigned to the half-spaces $x<0$ and $x>H$, respectively. Finally, let h_j, ϵ_j, μ_j and c_j, where $1\le j\le J$, be the thickness, permittivity, permeability and the wave speed of the j-th medium, so that $h_1=h_J=\infty$, and to agree with the notation in the previous sections, we assume that $\epsilon_J\equiv\epsilon_+$, $\mu_J\equiv\mu_+$, and $c_J\equiv c_+$.

It is shown in [7] that the reflection coefficient of the described structure has the value

$$R=\left|\frac{Z_{1,J-1}-Z_J}{Z_{1,J-1}+Z_J}\right| \quad \text{(A2.1)}$$

where Z_J and $Z_{1,J-1}$ are the impedances of the half-space $x>H$ and of the stack of $J-1$ layers in the domain $x<H$, which can be computed by the formulas described below. Thus, the impedance of the j-th layer is defined by the formula

$$Z_j=\begin{cases} Z_j^0/\cos\theta_j, & \text{for the }\perp\text{-polarization,}\\ Z_j^0\cos\theta_j, & \text{for the }\|\text{-polarization,}\end{cases} \quad \text{(A2.2)}$$

where

$$Z_j^0=\sqrt{\frac{\mu_j}{\epsilon_j}}, \quad \text{(A2.3)}$$

is a material parameter of the j-th layer, widely known as its wave resistance, [7], and θ_j is the angle of incidence in the j-th layer, which is related by the Snell law

$$\frac{\sin\theta_j}{c_j}=\frac{\sin\theta_J}{c_J}, \quad \text{(A2.4)}$$

with the angle of incidence $\theta_J = \theta_+$ in the right half-space $x > H$. Finally, the impedance of the stack of the first j layers can be computed by the recursive formulas

$$Z_{1,j} = Z_j \frac{Z_{1,j-1} - \mathrm{i}Z_j \tan(\omega h_j / c_j)}{Z_j - \mathrm{i}Z_{1,j-1} \tan(\omega h_j / c_j)}, \quad \ldots, \quad Z_{1,1} = Z_1, \tag{A2.5}$$

with the "initial value" corresponding to the degenerate structure consisting of two half-spaces.

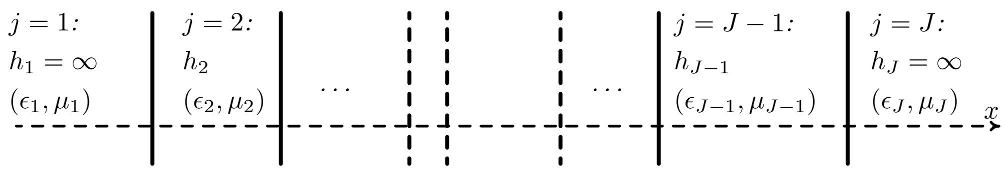


Figure 2: A layered structure

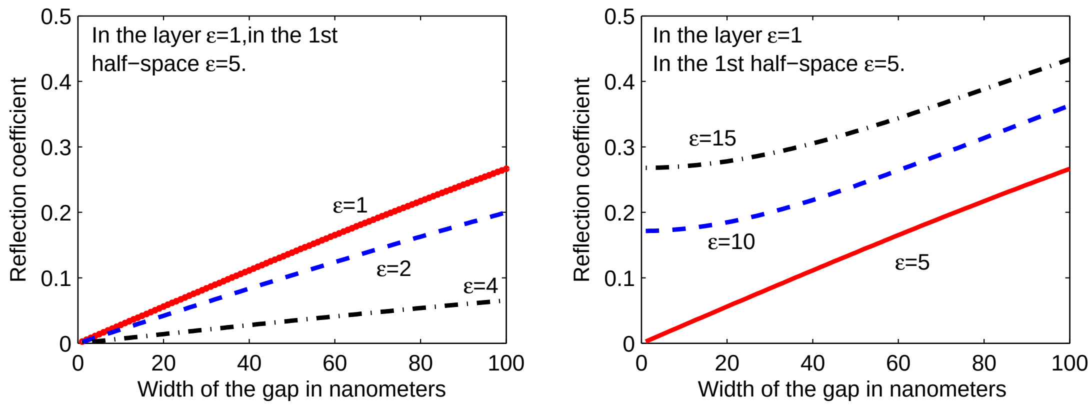
Although the above algorithm is valid for an arbitrary number of layers, it is instructive to apply it first to a simple structure consisting of two half-spaces of identical materials separated by a narrow layer of a different material. Indeed, the theory of wave propagation suggests that as the layer's width becomes negligible compared to the wavelength, the reflection coefficient of the structure should also become negligible. In other words, a very narrow layer should not affect thermal radiation, so that the thermal resistance of a layer of vanishing width should also vanish.

As an example we consider the reflection coefficient of a perpendicularly polarized electromagnetic wave with the wave length $\lambda = 2000\,\mathrm{nm}$ (in vacuum) propagating along the normals to the surfaces of two half-spaces separated by a narrow gap with a width below $100\,\mathrm{nm}$.

The left diagram in Fig. 3 illustrates the dependence of the reflection coefficient on the thickness of a layer between two half-spaces made from the same material with the permittivity $\epsilon = 5$, which corresponds to a glass. Three different curves in this figure correspond to gaps with permittivities $\epsilon = 1$ (vacuum), $\epsilon = 2$, and $\epsilon = 4$. The graphs clearly show that independently of layer's material, when its width decreases its reflection coefficient approaches zero proportionally to the layer's width as $H \to 0$. Therefore, since the reflected energy is proportional to the square of the reflection coefficient, it is natural to expect that the heat transfer coefficient of a vanishing layer increases proportionally to $1/H^2$, which agrees with recent experiments reported in [16] but disagrees with the claim made there that such result breaks known physical laws.

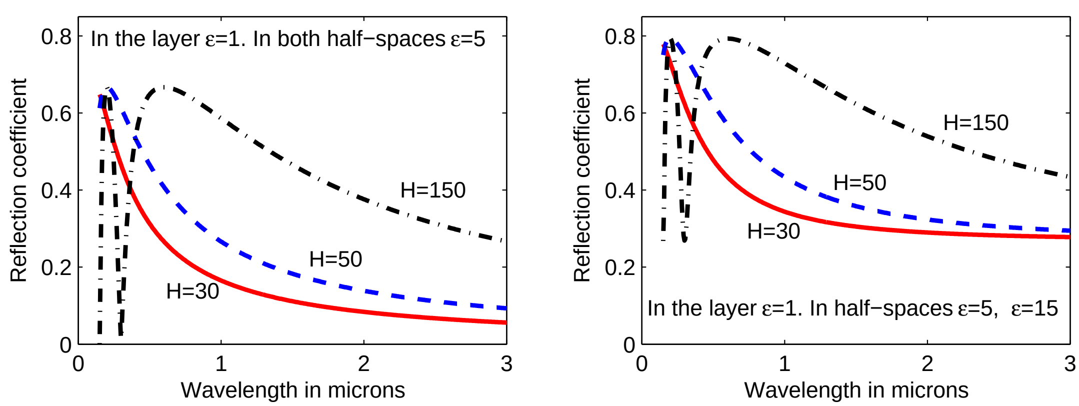
The right diagram in Fig. 3 illustrates the dependence of the reflection coefficient on the width of a vacuum gap ($\epsilon = 1$) between a glass half-space with the permittivity $\epsilon = 5$ and a half space with a different permittivity. The three graphs in this figure correspond to the permittivity of the second half-space set to $\epsilon = 5$, $\epsilon = 10$ and $\epsilon = 15$. These curves show that if the half-spaces are made from different materials then the reflection coefficient does not vanish even when the gap collapses to an interface. It also shows that a bigger contrast between the half-spaces results in the bigger reflection coefficient between them.

Fig. 4 illustrates the influence of the wavelength on the reflection coefficient of a narrow vacuum gap. As in the previous figure, the left graph in Fig. 4 corresponds to the cases when both half spaces have the same dielectric constant $\epsilon = 5$, and the right graph corresponds to the cases when the half-spaces have different permittivities, selected as $\epsilon = 5$, $\epsilon = 10$ and $\epsilon = 15$. Three graphs in



The left figure corresponds to $\epsilon = 5$ for both half-spaces and $\epsilon = 1$, 2 and 4 for the gap layer. The right figure corresponds to a gap layer with $\epsilon = 1$ between one half-space with $\epsilon = 5$ and the other half-space with $\epsilon = 5$, 10, and 15.

Figure 3: Dependence of the reflection coefficient on the thickness of a layer between two half-spaces for different layer materials and different half-space materials.



The left figure corresponds to epsilon = 5 for both half-spaces and $\epsilon = 1$, 2 and 4 for the gap layer. The right figure corresponds to a gap layer with $\epsilon = 1$ between one half-space with $\epsilon = 5$ and the other half-space with $\epsilon = 5$, 10, and 15.

Figure 4: Wavelength dependence of reflection coefficient of a layer between two half-spaces

each of the diagrams show the reflection coefficients of the gaps of widths $H = 30\,\text{nm}$, $H = 50\,\text{nm}$ and 150 nm. In all cases, the incident electromagnetic wave has a wavelength (in vacuum) from 150 nm to 3000 nm, and propagates along the normals to the interfaces with the perpendicular polarization.

Fig. 4 confirms that the reflection coefficient of the gap between similar materials is generally lower than the reflection coefficient of the gap between different materials. It also confirms the prediction that if the wavelength λ is at least three times longer than gap's width H, then the reflection coefficient of the gap monotonically decreases as the ratio λ/H increases. However, if $\lambda \lesssim 3H$, then the dependence of the reflection coefficient on the wavelength becomes more complicated, vanishing at certain frequencies and reaching noticeable levels at other frequencies. Such dependence of the reflection coefficient on the ratio H/λ of the gap's width to the wavelength has significant implications on the intensity of the radiative heat transport across a narrow gap. Indeed, thermally excited radiation has a broad spectrum whose important part has the wavelength spanning, at room temperature, from approximately one to twenty microns with a peak at about $\sim 5\,\mu$m. Therefore, for gaps narrower than $\sim 1\,\mu$m, the reflection coefficient of thermally excited radiation decays almost proportionally to the width H, and thus, as $H \to 0$, the heat transport should increase as $1/H^2$. However, the spectrum of thermal radiation at $3000\,^\circ$K spans from $\sim 0.2\,\mu$m to $\sim 30\,\mu$m, with the maximum at $\sim 0.9\,\mu$m. In this case, simple proportionality of the reflection coefficient to the width H can be expected only for gaps narrower than ~ 300 nm.

Next we consider a five-layer structure similar to that appearing in HAMR systems. These structures include a glass half-space ($\epsilon = 5$) covered by a 20 nm ferromagnetic layer ($\epsilon = 2$) with a 2 nm carbon overcoat ($\epsilon = 2$) and separated by an air gap from another half-space, which models a transducer.

The left diagram in Fig. 5 shows the dependence of the reflection coefficient of this structure on the material of the transducer. Thus, the bold solid line corresponds to the case when it is made from the same material as the base, i.e. with $\epsilon = 5$. The two graphs with higher reflection correspond to denser base materials with $\epsilon = 10$ and $\epsilon = 15$, and the two other graphs show the reflection coefficients in the cases with $\epsilon = 1.5$ and $\epsilon = 3$. Clearly, the reflection coefficient has a lower value when the stack of layers is surrounded by materials with similar optical properties. The influence of the wavelength on the reflective properties of different layered structures is also illustrated by the right diagram in Fig. 5, which plots the reflection coefficient against the wavelength for structures characterized by identical half-spaces ($\epsilon = 5$) and by the varying width of the air gap. The solid curve corresponds to 2 nm wide air gap, and the other curves correspond to 5 nm, 10, nm, 20 nm and 30 nm-wide gaps. It is easy to see that for thermal radiation at room temperature, the narrower gap within the considered range below 1 μm corresponds to lower reflection.

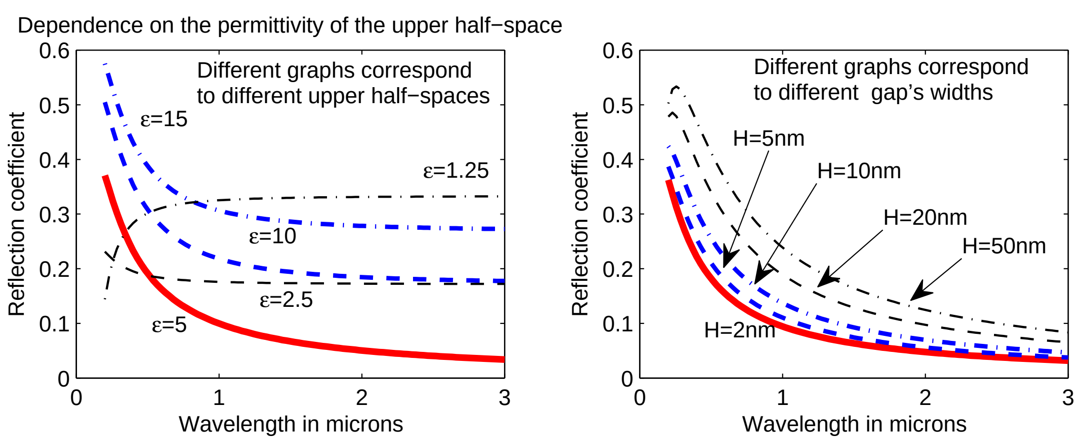


Figure 5: Reflection coefficients of a 5-layered structure similar to that in proposed HAMR systems