Optimal H_∞ Control Design and Implementation of Hard Disk Drives with Irregular Sampling Rates

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Abstract

Missing position error signal sampling data in hard disk drives (HDDs) results in their servo systems having irregular sampling rates (ISRs). With the natural periodicity of HDDs, which is related to the disk rotation, HDD servo systems with ISRs can be modeled as linear periodically time-varying systems. Based on our previous results, an explicit minimum entropy H_∞ *controller for HDD servos with ISRs via discrete Riccati equations is first obtained. The resulting controller is subsequently demonstrated to be periodically time-varying and implementable. Simulation and experimental studies, which have been carried out on a set of hard disk drives, demonstrate that the proposed control synthesis technique is able to handle ISRs and can be used to conveniently design a loop-shaping track-following servo that achieves the robust performance of a desired error rejection function for disturbance attenuation. Experimental results on ten actual 2.5" hard disk drives show around* 27% *improvement of the* 3σ *PES was obtained by the proposed control algorithm.*

1 Introduction

Since a hard disk drive (HDD) servo system with regular sampling rates can be modeled as a linear time-invariant (LTI) system [2] and the control theory for LTI systems is well developed, maintaining a regular sampling rate is quite attractive for HDD servos. However, sampling intervals for HDD servo systems are not always equidistant and even sometime an irregular sampling rate (ISR) due to missing position error signal (PES) sampling data is unavoidable. For example, false PES demodulation, due to incorrect servo address mark (SAM) detection [5] or damaged servo patterns in several servo sectors, makes the feedback PES unavailable in those servo sectors, resulting in an ISR. In this paper, the unavailability of feedback signals at a given sampling instance is referred to as a "missing sample". In addition, ISRs also frequently occur during the self-servo track writing (SSTW) process [1]. For example, during some SSTW processes, the time of writing final concentric servo patterns may coincide with the time of reading the feedback PES from previously written servo patterns. This conflict, due to the fact that HDD servo systems cannot read and write simultaneously, is referred to as a "collision" of reading PES with writing final servo patterns. Such a collision makes the feedback signal unavailable resulting in an ISR. Thus, it is important to address the issue of designing servo systems for HDDs under ISRs.

Generally speaking, the location of damaged servo sectors and the collision in the SSTW process is consistent on a given servo track. In other words, the unavailability of the PES for HDD servo systems occurs at some fixed locations for a given track. Furthermore, by considering that the natural periodicity of HDDs is related to the disk rotation, the servo systems can be represented by linear periodically time-varying (LPTV) systems with period equal to the number of servo sectors.

Since there tend to be large variations in HDD dynamics due to the variations in manufacturing and assembly, it is not sufficient to achieve adequate servo performance for a single plant. The designed controllers must guarantee a pre-specified level of performance for a large set of HDDs. By utilizing classical loop-shaping ideas which are familiar to most practicing engineers, the H_∞ loop-shaping control design [12] is well-suited to deal with the robust performance of the desired loop shape. Using such control, a set of HDDs with plant variations and a regular sampling rate can yield a desired error rejection function [6]. Thus, optimal H_∞ control is quite attractive for HDD servo systems. The purpose of this paper is to extend these results to HDDs with missing PES samples.

In this paper, we consider the optimal H_∞ control design for HDD servo systems with missing PES samples. These servo systems with missing samples are first modeled as LPTV systems, by considering the natural periodicity that is related to the disk rotation. Based on the results in [11], explicit and implementable solutions for the minimum entropy H_∞ control of HDD servos with missing PES samples were derived, by solving a set of discrete-time Riccati equations that are often significantly more computationally efficient and numerically robust than the solution of semi-definite programs (SDP) involving optimization problems constrained by linear matrix inequalities (LMIs) [3].

In order to demonstrate the effectiveness of the developed control algorithm, we apply it to the track-following control of multiple hard disk drives with missing PES samples. However, because the resulting controller is also periodically time-varying with period equal to the number of servo sectors, which is a large number for modern HDDs, a significant amount of memory is required to store all of the control parameters for the synthesized controller. Thus, in order to reduce the memory storage requirements, a simplified controller implementation is developed in this paper. Afterwards, the resulting optimal H_∞ track-following control is evaluated through both simulation and experimental study on multiple actual HDDs. The simulation study validates the effectiveness of the proposed control and the feasibility of the control parameter simplification. Experimental results on the ten tested HDDs validate the obtained robust performance.

This paper is organized as follows. Section 2 discusses the modeling of HDD servo systems with irregular sampling rates. Section 3 presents optimal H_∞ track-following control synthesis algorithm for HDDs with irregular sampling rates. The evaluation of the developed control synthesis algorithm on a set of actual HDDs is provided in Section 4. The conclusion is given in Section 5.

2 Modeling of HDD servo systems with irregular sampling rates

As mentioned in the previous section, an irregular sampling rate can be caused by missing a sample when the feedback PES is unavailable. Similarly to [8], HDD servo systems with missing PES samples can be modeled by the block diagram shown in Fig. 1. Here, we consider the output multiplicative uncertainty with the nominal plant of the voice coil motor (VCM) G_v^n and the plant uncertainty weighting function $W_\triangle$. In Fig. 1, d is the overall contribution of all disturbances [8] to PES. K is the controller to be designed by using the feedback signal $y(k)$ in the case of missing PES samples. Moreover, y is switched to PES when the PES is available, while y is switched to zero when the PES is unavailable [7].

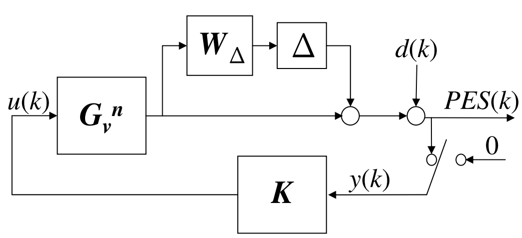

Figure 1: Modeling of HDD servo systems with an irregular sampling rate

Since the location of damaged servo sectors and the collision in the self-servo track writing process is fixed for each servo track, the unavailability of PES for HDD servo systems takes place at some fixed and pre-determined locations [1] on the disk for each track. Furthermore, with the natural periodicity of HDDs associated with the disk rotation, the servo systems can be represented by LPTV systems, which is presented in next section.

3 Optimal H_∞ control for HDD servo systems with irregular sampling rates

3.1 Optimal H_∞ control formulation

In order to deal with the HDD plant variations and missing PES samples, we will extend well-known error rejection loop-shaping design techniques for HDDs with regular sampling rates to HDDs with irregular sampling rates. For the optimal H_∞ control of HDD servo systems illustrated by the block diagram in Fig. 1, we need to design a controller K satisfying the following conditions [12] for the nominal VCM plant G_v^n:

$$\left\|T_s \cdot W_p\right\|_{2\leftarrow 2} < 1, \forall \|\triangle\|_\infty \leq 1 \tag{1}$$

where T_s is the sensitivity function (i.e. error rejection transfer function) from d to PES, as shown in Fig. 1, while W_p is the loop-shaping performance weighting function. Here, the H_∞ norm of linear time-invariant systems is generalized as ℓ_2 induced norm for LPTV systems [11]. Equation (1) illustrates the robust performance that the magnitude of the obtained error rejection function is always smaller than $\left|W_p(e^{j\omega})\right|^{-1}$ for the uncertain plant characterized by $W_\triangle$.

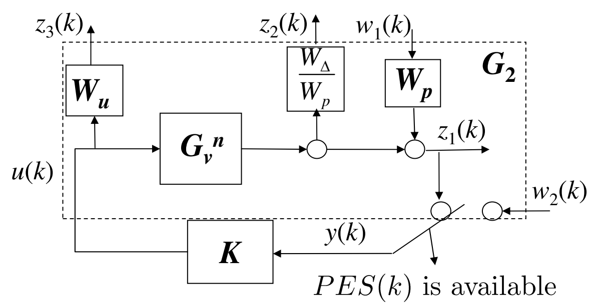

Figure 2: Control design formulation for HDDs with irregular sampling rates

Here, we consider the analogous block diagram to the optimal H_∞ control in [9] for the control design of the servo systems with an irregular sampling rate, as shown in Fig. 2. As discussed in [9], this control design formulation is different from the standard H_∞ control problem formulation [12] due to the introduction of the fictitious disturbance w_2 and the fact that the performance weighting function was moved to the disturbance input side. Suppose the open-loop LPTV system G_2 (with input $\begin{bmatrix} w_1 & w_2 & u \end{bmatrix}^T$ and output $\begin{bmatrix} z_1 & z_2 & z_3 & y \end{bmatrix}^T$) shown in Fig. 2 can be represented as the following state-space realization:

$$G_2 \sim \begin{bmatrix} x(k+1) \\ z(k) \\ y(k) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2(k) & D_{21}(k) & 0 \end{array}\right] \begin{bmatrix} x(k) \\ w(k) \\ u(k) \end{bmatrix} \tag{2}$$

with

$$\begin{bmatrix} C_2(k) & D_{21}(k) \end{bmatrix} = \begin{cases} \begin{bmatrix} C_{2m} & \begin{bmatrix} D_{21m} & 0 \end{bmatrix} \end{bmatrix} & \text{if } PES(k) \text{ is available} \\ \begin{bmatrix} 0 & \begin{bmatrix} 0 & 1 \end{bmatrix} \end{bmatrix} & \text{otherwise} \end{cases}$$

In (2), all of the matrix entries, except $C_2(k)$ and $D_{21}(k)$, are constant and $C_2(k)$ and $D_{21}(k)$ are periodic with period N.

From the standard assumption for optimal H_∞ control [11], the derived controller requires a condition $D_{21}(k)D_{21}^T(k) \succ 0$ for all k. With the control formulation illustrated in Fig. 2, the assumption that $D_{21}(k)D_{21}^T(k) \succ 0$ for all k is satisfied for this plant. In addition, it is known that the introduction of w_2 does not affect the optimal H_∞ controller and the optimal closed-loop ℓ_2 induced norm, which will be discussed in detail later.

With these changes and utilizing the block diagram in Fig. 2, the optimal H_∞ control problem for HDD servos with an ISR is to find an optimal linear time-varying controller K and a minimum γ with the closed loop ℓ_2 induced norm less than γ:

$$\begin{aligned} \min_{K,\gamma} \quad & \gamma \\ \text{s.t.} \quad & \|T_{z\leftarrow w}\|_{2\leftarrow 2} < \gamma \end{aligned} \tag{3}$$

where $T_{z\leftarrow w}$ represents the close-loop transfer function matrix from $w = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T$ to $z = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}^T$.

Throughout this paper, we use notations $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$, $D_{1\bullet} = \begin{bmatrix} D_{11} & D_{12} \end{bmatrix}$, $C(k) = \begin{bmatrix} C_1 \\ C_2(k) \end{bmatrix}$, and $D_{\bullet 1}(k) = \begin{bmatrix} D_{11} \\ D_{21}(k) \end{bmatrix}$.

3.2 Optimal H_∞ control synthesis algorithm

For the optimal H_∞ control synthesis for HDD servos with missing PES samples, we first need to obtain the unique minimum entropy controller satisfying the ℓ_2 induced norm constraint in (3) with a fixed γ, and then utilize a bi-section search method to find the minimum γ and the corresponding optimal controller.

In the minimum entropy control synthesis methodology that follows, for simplicity and without loss of generality, we will assume that $\gamma = 1$. For the system G_2 in (2), utilizing the control design algorithm presented in [8], yields the following unique stabilizing minimum entropy time-varying controller K, which satisfies the constraint in (3) and is given by the following state space realization:

$$\begin{cases} \hat{x}(k+1) = \bar{A}\hat{x}(k) + B_2 u(k) + F_t(k)\left(\bar{C}_{2m}\hat{x}(k) - y(k)\right) \\ u(k) = -T_{22}^{-1}\bar{C}_{12}\hat{x}(k) + L_t(k)\left(\bar{C}_{2m}\hat{x}(k) - y(k)\right) \end{cases} \tag{4}$$

where the parameters $\bar{A}$, $F_t(k)$, $\bar{C}_{2m}$, $T_{22}^{-1}\bar{C}_{12}$ and $L_t(k)$ are updated in the following steps:

1) Solve the state-feedback algebraic Riccati equation:

$$X = A^T XA + C_1^T C_1 - M\left(R + B^T XB\right)^{-1} M^T \tag{5}$$

where $R = D_{1\bullet}^T D_{1\bullet} - \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $M = A^T XB + C_1^T D_{1\bullet}$.

2) Compute $T_{11} \succ 0$, $T_{22} \succ 0$ and T_{21} using

$$R + B^T XB = \begin{bmatrix} T_{11} & 0 \\ T_{21} & T_{22} \end{bmatrix}^T \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} T_{11} & 0 \\ T_{21} & T_{22} \end{bmatrix}. \tag{6}$$

3) Get $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \left(R + B^T XB\right)^{-1} M^T$.

4) Calculate the following matrices for the filtering Riccati equation: $\bar{A} = A + B_1 F_1$, $\bar{C}_2(k) = C_2(k) + D_{21}(k)F_1$, and $\bar{C}_{12} = -T_{22}F_2$. Let $D_\perp$ be an orthogonal matrix of D_{12}. In addition, define a matrix W such that $W^T W = I - T_{11}^T T_{11}$ and W has appropriate dimensions so that the following matrix multiplication is well defined:

$$\begin{bmatrix} \bar{D}_{111} \\ \bar{D}_{112} \end{bmatrix} = D_\perp W + D_{12} T_{21}\ .$$

5) Update forwards in time the filtering Riccati equation solution with zero initial condition:

$$Y(k) = \bar{A}Y(k-1)\bar{A}^T + B_1 B_1^T - \tilde{M}(k) \times \left(\tilde{R}(k) + \begin{bmatrix} \bar{C}_{12} \\ \bar{C}_2(k) \end{bmatrix} Y(k-1) \begin{bmatrix} \bar{C}_{12} \\ \bar{C}_2(k) \end{bmatrix}^T\right)^{-1} \tilde{M}^T(k) \tag{7}$$

where $Y(k) \succeq 0$ and

$$\begin{aligned}
\tilde{M}(k) &= \bar{A}Y(k-1)\begin{bmatrix}\bar{C}_{12}\\ \bar{C}_2(k)\end{bmatrix}^T + B_1\begin{bmatrix}\bar{D}_{112}\\ D_{21}(k)\end{bmatrix}^T \\
\tilde{R}(k) &= \begin{bmatrix}\bar{D}_{112}\\ D_{21}(k)\end{bmatrix}\begin{bmatrix}\bar{D}_{112}\\ D_{21}(k)\end{bmatrix}^T - \begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}.
\end{aligned}$$

6) Compute $\tilde{T}(k) = \begin{bmatrix}\tilde{T}_{11}(k) & \tilde{T}_{12}(k)\\ 0 & \tilde{T}_{22}(k)\end{bmatrix}$, with $\tilde{T}_{11}, \tilde{T}_{22} \succ 0$:

$$\tilde{R}(k) + \begin{bmatrix}\bar{C}_{12}\\ \bar{C}_2(k)\end{bmatrix} Y(k-1) \begin{bmatrix}\bar{C}_{12}\\ \bar{C}_2(k)\end{bmatrix}^T = \tilde{T}(k)\begin{bmatrix}-I & 0\\ 0 & I\end{bmatrix}\tilde{T}^T(k) \tag{8}$$

7) Obtain

$$\begin{bmatrix}\tilde{F}_1(k)\\ \tilde{F}_2(k)\end{bmatrix} = \left(\tilde{R}(k) + \begin{bmatrix}\bar{C}_{12}\\ \bar{C}_2(k)\end{bmatrix} Y(k-1) \begin{bmatrix}\bar{C}_{12}\\ \bar{C}_2(k)\end{bmatrix}^T\right)^{-1} \tilde{M}^T(k).$$

8) Calculate the filter gains:

$$\begin{aligned}
L_t(k) &= T_{22}^{-1}\tilde{T}_{12}(k)\tilde{T}_{22}^{-1}(k) \\
F_t(k) &= \tilde{F}_1^T(k)\tilde{T}_{12}(k)\tilde{T}_{22}^{-1}(k) + \tilde{F}_2^T(k).
\end{aligned}$$

Taking into account the periodicity of G_2 in (2), we have the following Lemma.

Lemma 1. *For the LPTV system G_2 in* (2)*, the solution to the Riccati equation* (7) *is unique and periodic with period N. Furthermore, the H_∞ controller given by* (4) *is also periodic with period N.*

The proof of Lemma 1 was provided in [9]. The periodicity of H_∞ controllers for G_2 provides a significant advantage, since the Riccati equation (7) can be solved with zero initial conditions by iteration and the solution will converge to the corresponding periodic solution.

At this point, the optimal H_∞ control synthesis algorithm for the system in (2) is developed as follows.

Algorithm 1. *The following algorithm synthesizes optimal H_∞ control for the LPTV system G_2 in* (2)*.*

S1. Choose a large initial interval and a large initial value γ.

S2. For a given value γ, calculate the minimum entropy controller:

- *Solve the state feedback discrete algebraic Riccati equation* (5) *in Step 1.*
- *If X and the factorization in* (6) *exists, continue to solve the filtering Riccati equation in* (7) *with zero initial conditions by iteration to obtain $Y(k)\,(k = 0,\ldots,N-1)$. Otherwise, stop.*
- *If $Y(k) \succeq 0$ and the factorization in* (8) *exists for $\forall k = 0,\ldots,N-1$, continue to calculate the control parameters in* (4)*. Otherwise, stop.*

S3. If the interval is small enough, stop. Otherwise, update the interval and γ and then go back to S2.

Moreover, as demonstrated in [10], the filter gains $L_t(k)$ and $F_t(k)$ are equal to zero when $PES(k)$ is unavailable, which verifies the expectation that the minimum closed-loop ℓ_2 induced norm and the optimal H_∞ controller are unaffected by w_2 in the control synthesis. With the zero gains of $F_t(k)$ and $L_t(k)$ at the instance when the PES is unavailable, the time varying control parameter $\bar{C}_2(k) = C_2(k) + D_{21}(k)F_1$ in (4) can be simply substituted by a constant parameter $\bar{C}_{2m} = C_{2m} + \begin{bmatrix} D_{21m} & 0 \end{bmatrix} F_1$ without affecting the controller effect. Then, for the system in (2), all of the control parameters of the optimal H_∞ controller shown in (4) except $F_t(k)$ and $L_t(k)$ are constant.

4 Control synthesis evaluation on real HDDs

In order to evaluate our proposed optimal H_∞ control design methodology in this paper, the algorithm will be applied to the SSTW servo design of multiple HDDs with missing PES samples through both simulation and experimental studies. As illustrated in [1], based on the configuration of the SSTW, the set of servo sectors where the PES is unavailable can be pre-determined. Based on the configuration of the SSTW [1], we found there exist some "common" sets of missing-PES-sample servo sectors. In other words, the sequence of missing PES samples on different tracks is just the shifted version of one of these "common" sets. In addition, these "common" sets are quite similar to each other and each of the sets only has few different sectors from the others. Consequently, the controllers synthesized on different tracks that have different missing-PES-sample patterns, turn out to be very close to each other. It motivates us to utilize one "common" set to synthesize a single optimal H_∞ controller for all tracks that may have different missing-PES-sample patterns.

During the simulation and experimental studies on the SSTW servo design with missing PES samples, we consider the following procedures:

1. Construct the modeling of the VCM plant and determine a "common" set of missing-PES-sample servo sectors for the considered self-servo track writing servo.

2. Determine $W_{\triangle}$ based on the VCM modeling and choose W_p for desired disturbance attenuation.

3. Synthesize an optimal H_{∞} controller using Algorithm 1 in Section 3.

4. Simplify the synthesized controller, validate the controller simplification, and then implement the resulting controller on real HDDs.

The ten hard disk drives considered here were provided by Western Digital Corporation. For these 2.5" disk drives, the number of servo sectors is $N = 274$ and the spindle rotation speed is 9000 RPM. For the product type of the tested HDDs, we found a "common" set of missing PES servo sectors (including 57 servo sectors): M_{miss}={2, 6, 10, 14, 18, 27, 31, 35, 39, 43, 47, 52, 56, 60, 64, 68, 72, 81, 85, 89, 93, 97, 101, 110, 114, 118, 122, 126, 135, 139, 143, 147, 151, 155, 164, 168, 172, 176, 180, 184, 193, 197, 201, 205, 209, 218, 222, 226, 230, 234, 238, 247, 251, 255, 259, 263, 272 }. Such a missing sample sequence will be utilized to synthesize the optimal H_{∞} track-following controller in this section.

4.1 Plant identification and weighting function selection

For control synthesis convenience, notch filters are incorporated into the VCM plant G_v. In order to obtain the nominal VCM plant, the actual VCM plant frequency response for one single drive was measured. Then, the nominal VCM model $G_v^n(s)$ was identified to match the experiment frequency response, as described in [8]. As a result, the nominal VCM plant is just identified as a 7th order model.

The uncertainty weighting function $W_{\triangle}$ considered in the control design is shown in Fig. 3. The selected uncertainty weighting function demonstrates that the real plant could have an unstructured uncertainty of a $\pm 26\%$ gain variation at low frequency and a $\pm 112\%$ gain variation at high frequency respectively from the nominal plant.

The performance weighting function W_p is designed as a low-pass filter (W_p^{-1} is a high-pass filter) for disturbance attenuation. In this paper, the performance weighting function is determined as

$$W_p(z) = \frac{0.1594z^3 - 0.3437z^2 + 0.2469z - 0.05914}{z^3 - 2.966z^2 + 2.932z - 0.9662} \,. \tag{9}$$

Then, with the proposed control synthesis technique, all of the disk drives are expected to possess the better disturbance attenuation than the one described by the inverse of the performance weighting function shown in Fig. 5.

4.2 Control design and simplification

It is well known that if the optimal H_{∞} controller yields a minimum $\gamma^* \leq 1$, then the controller is able to achieve the robust performance by satisfying the corresponding conditions illustrated in (1). In other words, the designed

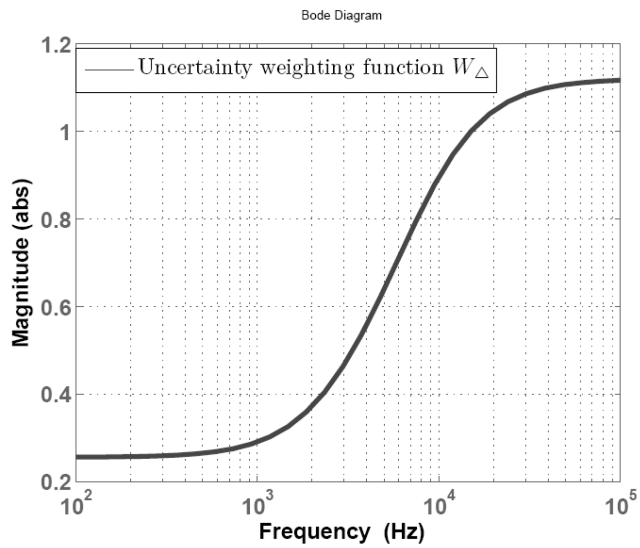

Figure 3: Plant uncertainty weighting function $W_\triangle$

controller achieves the better disturbance attenuation than the inverse of performance weighting function shown in Fig. 5 for all the plant variations with the plant uncertainty weighting function in (9). With the determined W_p and $W_\triangle$, the weighting value W_u is to be tuned so that the achieved γ is less than or equal to 1 and simultaneously the control actuation generated by the resulting controller K is appropriate under the hardware constraints of real HDD servo systems.

For the real hard disk drives described in the previous section, a weighting value of $W_u = 40$ was selected and thus the resulting optimal H_∞ control is able to achieve an optimal ℓ_2 induced norm $\gamma^* = 0.88$. The resulting gain $L_t(k)$ for one HDD revolution is shown in Fig. 4, verifying our prediction about the time-varying control parameters in Section 3.

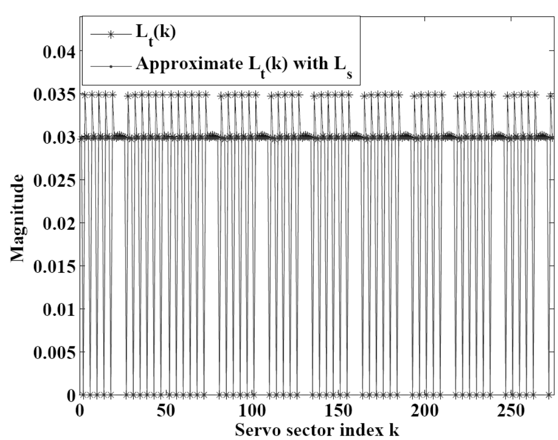

Figure 4: The designed $L_t(k)$

Since the control parameters $F_t(k)$ and $L_t(k)$ are time-varying, we have to save all 217 $(= 274 - 57)$ sets of non-

Table 1: Simulation Results

	3σ PES (% track)	
	Nominal	Worst
With the original control parameters $F_t(k)$ and $L_t(k)$	4.24	5.50
With the approximate control parameters using F_s and L_s	4.30	5.58

zero time-varying control parameters, requiring a significant amount of memory. Unfortunately, it is almost impossible to reserve so much memory for storing these control parameters in real HDDs. Fortunately, we found that the non-zero time-varying parameters $F_t(k)$ and $L_t(k)$ have very small variations, which motivates us to treat all non-zero time-varying control parameter values as constants. Specifically, the constant values F_s and L_s for $F_t(k)$ and $L_t(k)$ are obtained by taking the average of all non-zero $F_t(k)$ and $L_t(k)$ respectively. Then, the approximate control parameters F_s and L_s will be applied when the PES is available. Consequentially, just one set of control parameters F_s and L_s has to be stored in memory. The approximate values for $L_t(k)$ with L_s are also shown in Fig. 4. The simulation study, which will be presented in next section, shows that such control parameter approximation has a tolerable negative affect on the control performance.

4.3 Simulation study

In order to evaluate the robust performance of the designed controller, a total of 50 different VCM plants was collected. These various VCM plants were randomly generated based on the nominal VCM plant and the uncertainty weighting function shown in Fig. 3 by using Matlab function "`usample`". In addition, for each simulation on one generated VCM plant, a set of missing-PES-sample servo sectors is randomly choosen from the real missing-PES-sample sets to evaluate the controller synthesized using the "common" set M_{miss}.

The simulation results of the Root Mean Square (RMS) 3σ values of PES for the nominal plant and the worst-case plant are illustrated in Table 1. Based on these time-domain simulation results, we are confident that the proposed optimal H_∞ control indeed attains its predicted robust performance. Moreover, Table 1 also shows the simulation results by using the approximate control parameters. The results imply that the performance degradation caused by the control parameter approximation is so small that the control simplification presented in the previous section is viable.

4.4 Control implementation study

The simulation results presented in the previous section illustrate that the proposed control design methodology in this paper is able to achieve robust performance under an ISR. In this subsection, we discuss its implementation on a set of actual SSTW servos with missing PES samples. The designed controller was coded on the disk drives' own processor by changing their firmware code. In the experimental study, the designed controller was evaluated on different tracks having different missing-PES-sample patterns from M_{miss} that was utilized to synthesize the controller.

Unlike LTI systems, a time-varying system with irregular sampling rates has no well-defined error rejection transfer function. However, in order to evaluate the implemented controller, we define the following disturbance-PES relationship called approximate error rejection transfer function. For such an approximate error rejection function, we still measure the frequency response from d to PES shown in Fig. 1. However, when PES is unaccessible at the time when missing samples occur, we approximately treat the previous available PES as the current unavailable PES. The approximate error rejection transfer functions were measured on the ten considered disk drives and are shown as the black lines in Fig. 5.

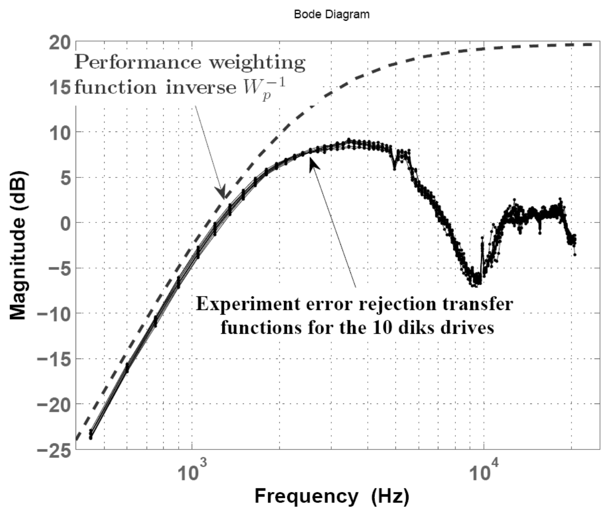

Figure 5: Experiment result for approximate error rejection functions

The inverse of the performance weighting function, indicated by the red line, is shown in Fig. 5 as well. The experiment results in Fig. 5 illustrate that all of the measured approximate error rejection transfer functions are below the inverse of the performance weighting function at every frequency. Thus, the obtained experiment results on multiple real disk drives demonstrate that the optimal H_∞ track-following control synthesized by our proposed control algorithm achieves the robust performance of the desired disturbance attenuation.

Figure 6 shows the histogram of the 3σ values for the closed-loop NRRO PES with the implemented controller on the tested disk drives. Notice that these experimental results for the closed-loop PES are similar to the simulation results listed in Table 1, which further validates our optimal H_∞ control design for the HDDs with missing PES samples.

In the experimental study, we also compared the control synthesis algorithm presented in this paper to a reference control design that uses the previous available one when a missing PES sample occurs. The corresponding experimental results are also shown in Fig. 6. We found that the controller designed using our proposed optimal H_∞ control synthesis algorithm improves 3σ PES by around 27% in the experimental study.

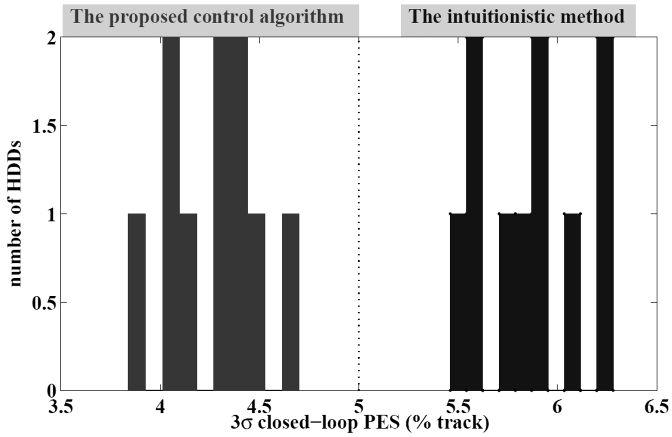

Figure 6: The histogram of 3σ closed-loop PES values for the ten tested HDDs using the proposed control algorithm and the intuitionistic method

4.5 Discussion

As demonstrated in the simulation and experimental studies, the proposed controller is able to significantly improve the servo performance of the SSTW compared to the referred intuitionistic methodology. The major advantage of our proposed H_∞ control synthesis algorithm is that the resulting controller can produce a desired robust performance for HDDs with missing PES samples and improve disturbance attenuation by choosing an appropriate performance weighting function. As discussed at the beginning of this section, for SSTW servos, we just need one single "common" set M_{miss} to synthesize one single controller for different tracks and different HDDs. Through the simulation study, the synthesized controller can be further simplified for the implementation on real HDDs.

5 Conclusion

In hard disk drive servo systems, sometimes an irregular sampling rate is unavoidable, for example, when false PES demodulation is caused by damaged servo sectors and when the unavailability of feedback signals is due to the collision in self-servo track writing process. In this paper, by considering the natural periodicity of HDDs due to the disk rotation, the servo systems with missing PES samples were represented by LPTV systems with period equal to the number of servo sectors. Using the modeled LPTV system, we studied optimal H_∞ track-following control synthesis for HDDs with missing PES samples so as to guarantee a pre-specified level of performance for these HDDs having large dynamics variations. The simulation and experimental study on multiple hard disk drives demonstrated the developed algorithm's effectiveness in handling irregular sampling rates and achieving the robust performance of a desired error rejection transfer function. In the experimental study, around 27% improvement of the 3σ PES was obtained by the proposed control algorithm over the intuitionistic methodology for the ten tested disk drives.

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