A Phenomenological Heat Transfer Model for the Molecular Gas Lubrication System in Hard Disk Drives

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Abstract

A reliable model of the heat transfer at the air bearing surface of the flying head slider is important for treating thermomechanical aspects of the molecular gas lubrication system in hard disk drives. This paper proposes a new model for heat transfer in the head disk interface, which considers both the heat conduction and viscous dissipation. The conduction heat flux based on this model shows better agreement with numerical results of the linearized Boltzmann equation than existing models derived from the temperature jump theory. The viscous dissipation of plane Couette flow as well as that of plane Poiseuille flow in the gas film is analyzed using the energy conservation equation instead of the linearized Boltzmann equation, which is incapable of calculating the viscous dissipation at the boundaries. The new model gives simple analytical expressions for the heat flux contributed by heat conduction and viscous dissipation, and it can be easily applied to numerical thermomechanical simulations of the slider's performance.

I. INTRODUCTION

Heat transfer and thermomechanical issues become more important to the molecular gas lubrication system in hard disk drives as the flying heights of air bearing sliders reduce to one or two nanometers in order to achieve higher magnetic recording densities. In the read/write head of a slider, the write current not only induces thermal disturbances to the read-back signal, due to the temperature-dependence of the read head's magnetoresistance[1], but also causes a thermal deformation of the slider's air bearing surface (ABS), resulting in a greater possibility of slider-disk impact. Recently a thermal actuation technology[2], known as thermal flying height control (TFC) or dynamic fly height (DFH), has been developed to make use of the thermal protrusion of the head pole tip to achieve a lower flying height and a higher recording density of hard disk drives. In this technology, a micro heating element embedded in the slider body controls the protrusion and slider-disk gap through its input Joule heating current. The heat transfer between the slider and the disk affects the temperature distribution inside the slider and the deformation of the slider at the trailing edge. Thus, an accurate description of the heat transfer on the ABS is important to numerical analyses of the slider's performance.[3]

Several models for the heat transfer in the head disk interface (HDI) have been proposed to calculate the heat flux on the slider's air bearing surface for the simulation of either the read head temperature or the flying attitude of a thermally actuated slider. Zhang and Bogy[4] solved the reduced Navier-Stokes equation with velocity slip and temperature jump boundary conditions and obtained an analytical formula for the heat transfer in the HDI. Chen *et al.*[5] extended Zhang and Bogy's work by including the work done by the pressure gradient. Simulations based on their new formula compare well with experimental results.[5] Since both of these approaches are based on the slip flow theory, it is not guaranteed that these formulae are applicable for transition flows or free molecular flows. Ju[6] used the direct simulation Monte Carlo (DSMC) method to numerically analyze the heat conduction and viscous dissipation induced by plane Couette flow. His results showed that the heat conduction model based on the temperature jump theory holds even for a highly rarefied gas. For the viscous dissipation contributed by Couette flow, he proposed an empirical formula that fits well with his DSMC results. Since the heat transfer in the HDI is also contributed by other mechanisms than those discussed by Ju, Shen and Chen[7] started with a linearized BGK-Boltzmann equation to analyze this problem. They showed that the heat transfer can be essentially divided into two parts: heat conduction and viscous dissipation due to the Couette and Poiseuille flows. Their results are, however, not fully consistent with the results based on the linearized Boltzmann equation, which shows that viscous dissipation vanishes in the flow.

In this paper, we separately analyze the heat conduction and viscous dissipation using different approaches. For the heat conduction, we modify the mean free path by including the effects of the boundaries, i.e., the slider and the disk. Our results agree better with the numerical calculations based on the linearized Boltzmann equation. For the viscous dissipation, we adopt an approach that is different from that used by previous researchers. Here, we instead work with the conservation equations and intrinsic symmetry of plane

Couette and Poiseuille flows and obtain analytical formulae for viscous dissipation at the boundaries in both flows.

II. CONDUCTION HEAT FLUX MODEL

The first attempt to model the heat conduction in the gas lubrication film between a slider and a disk, as shown by Fig. 1, used the temperature jump boundary condition in solving the energy equation.[4, 5, 6] This boundary condition states that the temperature jump between a boundary plate and the gas at the boundary is proportional to the local temperature gradient,

$$T - T_w = \frac{(2-\sigma_T)}{\sigma_T}\frac{2\gamma}{(\gamma+1)}\frac{1}{\Pr}\mathrm{Kn}\frac{\partial T}{\partial n}, \tag{1}$$

where T_w, σ_T and n are the temperature, the thermal accommodation coefficient and the unit outer normal of the boundary plate, respectively; Pr is the Prandtl number, γ is the specific ratio and T is the temperature of the gas at the boundary; Kn is the Knudsen number, which is defined as the ratio of the mean free path of gas molecules λ to the gas film thickness h, i.e. Kn=λ/h. Through this approach, Zhang and Bogy[4], among others, showed that the conduction heat flux at the bearing surface is,

$$q_{con}\Big|_{\text{bearing surface}} = -k\frac{T_s - T_d}{h + 2\dfrac{(2-\sigma_T)}{\sigma_T}\dfrac{2\gamma}{(\gamma+1)}\dfrac{1}{\Pr}\lambda}. \tag{2}$$

where k is the gas thermal conductivity, and T_s and T_d are the temperatures of the upper and lower boundary plates, i.e. the slider and disk. When the boundary plates are fully diffused, i.e. $\sigma_T = 1$, the non-dimensional heat conduction flux can be written as[7],

$$Q_{con}\Big|_{\text{bearing surface}} = \frac{q_{con}\Big|_{\text{bearing surface}}}{\rho_0\sqrt{2RT_0}\dfrac{T_s - T_d}{T_0}} = \frac{15}{16D + 28\sqrt{\pi}}. \tag{3}$$

where ρ_0 is the temperature, T_0 is the ambient gas density, R is the specific gas constant and $D = \sqrt{\pi}/(2Kn) = \sqrt{\pi}h/(2\lambda)$ is referred to as the inverse Knudsen number. For a general σ_T less than 1, the non-dimensional heat conduction flux is,

$$Q_{con}\Big|_{\text{bearing surface}} = \frac{15}{16D + \dfrac{2-\sigma_T}{\sigma_T}28\sqrt{\pi}}. \tag{4}$$

It is seen that the mean free path λ is an important parameter in the temperature jump boundary condition and the conduction heat flux. In kinetic theory, the mean free path is defined as the average distance traveled by gas molecules between two collisions at the equilibrium state, where there is no presence of boundary plates. It is a function of the gas temperature and pressure. It can be seen that the free distance traveled by a gas molecule is reduced if that molecule is close to a boundary plate. The effect of the boundary plate(s) on the free path of gas molecules needs to be considered. Such a modified mean free path appears to be more applicable to the temperature jump theory.

The modified mean free path of gas molecules due to two parallel boundary plates is calculated in two steps. First, the free path distance of one molecule affected by one or two plates is calculated, respectively. Figure 2(a) shows the case of one molecule close to one plate. Here it is assumed that the molecule's collision with another molecule is almost sure to happen when it travels a distance λ. This means that only when the distance d between the molecule and the boundary is less than λ will its mean free path be affected by the presence of the plate. A second assumption is that the velocity directions of molecules are uniformly distributed in the 3-D space. For the molecule in Fig. 2(a) with a distance $d < \lambda$ from the boundary, when the angle θ between the molecular velocity and the normal of the boundary is larger than arccos(d/λ), the molecule's free path distance is still λ. When the angle θ is less than arccos(d/λ), the molecule's free path distance becomes $d/\cos(\theta)$. The corresponding possibility of the molecule's collision with the boundary at an angle θ and in a solid angle $d\omega$, where $d\omega=\sin(\theta)d\theta d\varphi$ with φ denoting the azimuthal angle, is $\sin(\theta)\, d\theta/2$. Then the mean value of the free path of that molecule is,

$$\lambda_1 = \int_0^{\arccos(d/\lambda)} \frac{d}{\cos(\theta)}\frac{\sin(\theta)}{2}d\theta + \int_{\arccos(d/\lambda)}^{\pi} \lambda\frac{\sin(\theta)}{2}d\theta = \frac{\lambda}{2}[1+\frac{d}{\lambda}-\frac{d}{\lambda}\ln(\frac{d}{\lambda})]\,. \quad (5)$$

Similarly, the mean free path λ_2 of one molecular affected by two boundaries in Fig. 2(b), can be obtained as,

$$\begin{aligned}\lambda_2 &= \int_0^{\arccos(\frac{d}{\lambda})} \frac{d}{\cos(\theta)}\frac{\sin(\theta)}{2}d\theta + \int_{\arccos(\frac{d}{\lambda})}^{\arccos(\frac{h-d}{\lambda})} \lambda\frac{\sin(\theta)}{2}d\theta + \int_{\arccos(\frac{d}{\lambda})}^{\pi} \frac{h-d}{\cos(\theta)}\frac{\sin(\theta)}{2}d\theta \\ &= \frac{\lambda}{2}[\frac{h}{\lambda}-\frac{d}{\lambda}\ln(\frac{d}{\lambda})-\frac{h-d}{\lambda}\ln(\frac{h-d}{\lambda})]\end{aligned} \quad (6)$$

Second, the modified mean free path of the gas film is taken as the average value of the mean free paths of all molecules in the gas film. Among all of the gas molecules between two boundary plates, some of them are affected by one boundary if their positions satisfy $(d-\lambda)(h-d-\lambda) < 0$, some of them are affected by two boundaries if their positions satisfy $d < \lambda$ and $h-d < \lambda$ and the others, which satisfy $d > \lambda$ and $h-d > \lambda$, are not affected by any boundary. Here we assume that the gas molecules are uniformly distributed between the two boundaries. After some algebra in the mean calculation, the modified mean free path of the gas molecules between two boundary plates can be written as,

$$\lambda_m = \begin{cases} \lambda(1 - \frac{1}{4}\frac{\lambda}{h}), & h \geq \lambda \\ \lambda(\frac{3}{4}\frac{h}{\lambda} - \frac{h}{2\lambda}\ln(\frac{h}{\lambda}), & h < \lambda \end{cases} \tag{7}$$

Although this result is similar to that obtained by Peng *et al.*[8], the main difference lies in that we consider the effect of the two boundaries at the same time while Peng *et al.* only considered one of them.

With this modified mean free path, we can define a modified Knudsen number and a modified inverse Knudsen number as,

$$\mathrm{Kn}_m = \frac{\lambda_m}{h} = \begin{cases} 1 - \frac{\mathrm{Kn}}{4}, & \mathrm{Kn} \leq 1 \\ \frac{3\mathrm{Kn}}{4} - \frac{1}{2\mathrm{Kn}}\ln(\frac{1}{\mathrm{Kn}}), & \mathrm{Kn} > 1 \end{cases} \tag{8}$$

$$D_m = \frac{\sqrt{\pi}}{2}\frac{h}{\lambda} = \begin{cases} 1 - \frac{\sqrt{\pi}}{8D}, & D \geq \frac{\sqrt{\pi}}{2} \\ \frac{3D}{2\sqrt{\pi}} - \frac{D}{\sqrt{\pi}}\ln(\frac{2D}{\sqrt{\pi}}), & D < \frac{\sqrt{\pi}}{2} \end{cases} \tag{9}$$

A new heat conduction model is now proposed with the consideration of the effect of two parallel boundary plates on the gas mean free path. When the modified mean free path takes the place of the original mean free path in the temperature jump boundary condition, the temperature jump and non-dimensional conduction heat flux keep their original forms in Eqs. (1) and (4) with Kn and D replaced by Kn_m and D_m, respectively. It is important to emphasize that the modified mean free path is unnecessary for a gas film heat conduction model based on the Boltzmann equation since the mean free path in the Boltzmann equation with the BGK model or any other model is a characteristic value of the gas molecule model and the boundary effects are included there as the boundary conditions.

The new gas film heat conduction model still relies on the temperature jump theory, and it is not directly proved from kinetic theory. However, it is in good agreement with the model based on the linearized Boltzmann equation by Bassanini et al.[9]. Figure 3 shows the dependence of the non-dimensional heat conduction flux on the inverse Knudsen number D in Colors blue, red and green for the thermal accommodation coefficients of the boundary plates having the values 1, 0.826 and 0.759, respectively. For different thermal accommodation coefficients, results based on the new model (solid lines) agree well with Bassanini's results obtained through a variational approach for the linearized Boltzmann equation with the BGK model[9] (triangles). As shown by Fig. 3, the results based on the original definition of the mean free path[4] overpredict the heat flux, especially for inverse Knudsen numbers $D < 1$.

One important point to be noted is that this new model, which is based on the temperature jump theory and the modified mean free path, is very simple and easy to implement in any practical simulation of thermomechanical issues of the gas lubrication system in thermally actuated sliders.

The temperature dependence of the mean free path was taken into consideration by

Zhou et al.[10] Their approach can be viewed as another way of modifying the mean free path. In their approach the variable soft sphere model[11] is used to calculate the mean free path for the heat conduction model base on the temperature jump theory. Their modified mean free path is expressed as,

$$\lambda_{T,VSS} = \xi (\frac{T}{T_0})^{\omega+0.5} \lambda_{T_0,HS}, \tag{10}$$

where the parameters ξ and ω are recommend to be in the range 0.80-0.85 and approximately 0.75 for the air film, respectively, and $\lambda_{T_0,HS}$ is the mean free path of hard-sphere molecules at the reference temperature T_0.

Figure 4 shows a comparison of the conduction heat flux based on the modified mean free path model developed here with the heat flux based on the mean free path in Eq. (10). Here T_0 = 288 K and σ_T = 1. The non-dimensional heat flux based on Zhang and Bogy's model, Eq. (3)[4] (dashed line), is close to that based on Zhou's model[10] for $T = T_0$ (dash-dot line) and $T = T_0 + 80$ K (dotted line). Compared with the new model presented here (solid line), both Zhang and Bogy's and Zhou's model overestimate the heat flux, especially when D is less than 1. Thus we can conclude that the temperature effect on the non-dimensional heat flux is much smaller than the effect caused by the modification of the mean free path in Eq. (7).

III. VISCOUS DISSIPATION

The preferred way to address the viscous dissipation in a rarefied gas is to use the full Boltzmann equation, but it is complex and makes analysis formidable. Since the gas velocity in our case is much smaller than the molecular thermal velocity, the Boltzmann equation can be linearized. This is essentially the approach adopted by Shen and Chen[7]. In view of the discussion of the similarity solution by Sone[12], the gas flow with heat transfer in the HDI, when analyzed in the framework of the linearized Boltzmann equation, can be divided into two separate problems: heat conduction between two boundary plates at different temperatures and viscous flow between two plates at the same temperature. The first problem has been studied in section 2. We now turn our attention to the second one and discuss the application of the linearized Boltzmann equation to this problem.

The relation between a general gas molecular velocity distribution f and the Maxwell distribution f_0 of its equilibrium state at rest is written as[13],

$$f=f_0(1+\Phi), \tag{11}$$

where Φ is viewed as a non-dimensional velocity distribution function, i.e. $\Phi = f/f_0 - 1$. As an approach to the analysis of gases that deviate slightly from the equilibrium state at rest, the linearized Boltzmann equation neglects all of the nonlinear terms of Φ[13]. Following this linearization, the nonlinear terms in Φ are also neglected in all of the expressions of macroscopic physical variables in terms of the velocity distribution function[13]. Then, the non-dimensional heat transfer flux Q_y is reduced to,

$$Q_y = \iiint \xi_y (\xi_x^2 + \xi_y^2 + \xi_z^2)\phi\pi^{-3/2} \exp(-\xi_x^2 - \xi_y^2 - \xi_z^2) d\xi_x d\xi_y d\xi_z - \frac{5}{2}U_y , \tag{12}$$

where ξ_i is the non-dimensional molecular velocity and U_y is the non-dimensional macroscopic velocity. In kinetic analyses of a molecular gas lubrication film, it is always assumed that the flow velocity in the film thickness direction is negligibly small[7,14], resulting in an approximation $U_y = 0$. Using the linearized Boltzmann solution of ϕ with the BGK model for the molecular gas lubrication[7, 14], it can be shown that the first term in Eq. (12) vanishes as well. Hence, the heat flux $Q_y = 0$ for the molecular gas lubrication film between two boundary plates at the same temperature. This means that the BGK-linearized Boltzmann equation gives a zero heat flux perpendicular to the boundary when viscous dissipation is considered in the gas lubrication film. The same conclusion is arrived at when the linearized Boltzmann equation with the hard-sphere molecular model is solved[12].

Therefore, in order to analyze the heat transfer in the head disk interface, we need to go beyond the linearized Boltzmann equation and work with the full Boltzmann equation instead. Even with the BGK model, the full Boltzmann equation is formidable to analyze. Here, we propose to use instead the conservation equations, which are satisfied by the full Boltzmann equation, together with some intrinsic properties of plane Couette and Poiseuille flows to address the viscous dissipation problem. A treatment from another point of view will be presented in a separate paper.

i) Viscous dissipation due to Couette flow

Here we analyze the viscous dissipation in a Couette flow between two boundary plates at the same temperature. We again make the assumptions that the flow velocity in the thickness direction of the gas film is negligible and that the film thickness is much smaller than its length. So the gas flow and heat transfer are expected to have no change with respect to the x-direction.

Figure 5 shows a plane Couette flow, two parallel boundary plates and the coordinate system. In this coordinate system, the upper boundary is moving to the left with a speed $u_0/2$ and the lower boundary is moving to the right with the same speed. Here u_0 is assumed to be much smaller than the molecular thermal speed, which is the case in the head disk interface. The plane Couette flow is skew-symmetric with respect to its center line in the x-direction of this coordinate system. The choice of this particular coordinate system has no effect on the calculation of heat flux, since the flow is subsonic.

Due to the slip at the boundary when the gas is rarefied, the gas next to the boundary moves relative to it and thus the friction force does work and induces energy dissipation as well. Thus, we here distinguish two cases, as shown by Figs. 5(b) and 5(c). In the first one, we focus on the viscous dissipation in the Couette flow with no boundary plates. In the second one, we include the boundary plates and discuss the total viscous dissipation.

Let us consider the first case with no boundary plates. A control volume across the flow boundaries with length dx is set up, as shown by Fig. 5(b). According to kinetic theory, the integral form of the steady state energy conservation equation for this control volume is,

$$\begin{aligned}&\int_0^h \frac{\partial}{\partial x}[\rho u_x(\eta+\frac{u_x^2+u_y^2}{2})]dy+\int_0^h \frac{\partial}{\partial y}[\rho u_y(\eta+\frac{u_x^2+u_y^2}{2})]dy\\&=\int_0^h \frac{\partial}{\partial x}(\tau_{xy}u_y-q_x)dy+\int_0^h \frac{\partial}{\partial y}(\tau_{xy}u_x-q_y)dy\end{aligned}, \qquad (13)$$

where η is the gas enthalpy per unit mass, ρ is the gas density, q_i is the heat flux, u_i is the flow velocity ($i = x$, y or z) and τ_{xy} is the shear stress in the gas. Notice that $u_y = 0$ under our assumption. The plane Couette flow does not change in the x-direction, so all of the derivatives with respect to x are zero. Then, both of the terms on the left hand side and the first term on the right hand side of Eq. (13) vanish. Due to the skew-symmetry of the Couette flow with respect to its center line, the heat flux q_y, the flow velocity u_x and the shear stress τ_{xy} have the same magnitude at the upper and lower flow boundaries. In the coordinate system as shown by Fig. 5(a), we have $q_y|_{y=0} = -q_y|_{y=h}$, $u_x|_{y0} = -u_x|_{y=h}$ and $\tau_{xy}|_{y=0} = \tau_{xy}|_{y=h}$. Using these relationship in Eq. (13), we arrive at,

$$q_{y|y=h} = \tau_{xy|y=h}\, u_{x|y=h}\,. \tag{14}$$

As a check of Eq. (14), we consider two limit cases: slip flows and free molecular flows. For slip flows, $\tau_{xy}\Big|_{y=h} = -\dfrac{\mu u_0}{h + 2\dfrac{(2-\alpha)}{\alpha}\lambda}$ and $u_x\Big|_{y=h} = -\dfrac{u_0}{2}\dfrac{h}{h + 2\dfrac{(2-\alpha)}{\alpha}\lambda}$, where α denotes the surface momentum accommodation coefficient of the boundary plate. The heat flux given by Eq. (14) is then written as $\dfrac{\mu {u_0}^2 h}{2[h + 2\dfrac{(2-\alpha)}{\alpha}\lambda]^2}$ and it is exactly the result obtained by Zhang and Bogy[4].

For free molecular flows, $u_x = 0$, and Eq. (14) gives a zero heat flux at the gas boundaries. In kinetic theory, the velocity distribution function of gas molecules for free-molecular Couette flow is[15],

$$
f = \begin{cases} (\frac{1}{2\pi RT_0})^{\frac{3}{2}} \exp\{-[(c_x - \frac{u}{2})^2 + c_y^2 + c_z^2] \frac{1}{2RT_0}\} & , c_y < 0 \\ (\frac{1}{2\pi RT_0})^{\frac{3}{2}} \exp\{-[(c_x + \frac{u}{2})^2 + c_y^2 + c_z^2] \frac{1}{2RT_0}\} & , c_y > 0 \end{cases} \tag{15}
$$

where c_i is the molecular velocity in the i-direction with $i = x$, y or z. The macroscopic heat flux q_i is

$$
q_i = \int_{c_x}\int_{c_y}\int_{c_z} \frac{1}{2}\rho(c_i - u_i) \sum_{j=x,y \text{ and } z} (c_j - u_j)^2 f dc_x dc_y dc_z \ , \tag{16}
$$

Using Eq. (15) in Eq. (16), we get

$$
\begin{aligned}
q_y\big|_{y=h} &= \int_{c_x}\int_{c_y}\int_{c_z} \frac{1}{2} mnc_y (c_x^2 + c_y^2 + c_z^2) f dc_x dc_y dc_z \\
&= \frac{mn}{2}\{\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{\infty}^{\infty} c_y (c_x^2 + c_y^2 + c_z^2)(\frac{1}{2\pi RT_0})^{\frac{3}{2}} \exp\{-[(c_x - \frac{u_0}{2})^2 + c_y^2 + c_z^2]\frac{1}{2RT_0}\} dc_x dc_y dc_z \\
&\quad + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{\infty}^{\infty} c_y (c_x^2 + c_y^2 + c_z^2)(\frac{1}{2\pi RT_0})^{\frac{3}{2}} \exp\{-[(c_x + \frac{u_0}{2})^2 + c_y^2 + c_z^2]\frac{1}{2RT_0}\} dc_x dc_y dc_z\} \\
&= 0,
\end{aligned} \tag{17}
$$

which is the same as that obtained from Eq. (14).

In the second case where the dissipation due to the relative motion of gas on the boundaries is included, we change to another control volume that includes the upper and lower plates as shown by Fig. 5(c). The plates are assumed to be thin and have a constant temperature. Under the approximation $u_y = 0$, the integral form of the energy conservation equation can be written as,

$$
\begin{aligned}
&\int_0^h \frac{\partial}{\partial x}[\rho u_x (\eta + \frac{u_x^2 + u_y^2}{2})] dy \\
&\quad = [\int_0^h \frac{\partial}{\partial x}(-q_x) dy - (q_y\big|_{upper} - q_y\big|_{lower})] + [\tau\big|_{upper}(-\frac{u_0}{2}) - \tau\big|_{upper}\frac{u_0}{2}]
\end{aligned} \ , \tag{18}
$$

where $\tau|_{upper}$ and $\tau|_{lower}$ are the external shear stresses acting on the upper and lower thin plates to balance the flow friction, and $q_y|_{upper}$ and $q_y|_{lower}$ are the y-components of the heat flux vector on the upper and lower plates, respectively. The left hand side of Eq. (18) represents the net flux of the thermal and kinetic energy. The first bracket on the right hand side of Eq. (18) represents the heat flowing into the control volume and the second represents the work done by external forces. Due to the skew-symmetry of the boundary-plate-and-Couette-flow system with respect to the center line, it is still valid that $q_y|_{upper} = -q_y|_{lower}$. Similar to the first case, all the derivatives with respect to x vanish. We note that the external shear stresses satisfy $\tau|_{upper} = \tau_{xy}|_{y=h}$ and $\tau|_{lower} = \tau_{xy}|_{y=0}$. Using all these in Eq. (18), we get

$$q_y\Big|_{upper} = \tau_{xy}\Big|_{y=h}(-\frac{u_0}{2})\,. \tag{19}$$

This is the heat flux at the upper boundary plate.

Based on the method of moments, Liu and Lees[16] obtained the shear stress on the upper boundary in a Couette flow between two diffusely reflected boundary plates as,

$$\tau_{xy}\Big|_{y=h} = -\rho u_0\sqrt{\frac{2RT_0}{\pi}}\frac{\lambda}{d+2\lambda} \tag{20}$$

Then Eq. (19) becomes

$$q_y\Big|_{upper} = \frac{1}{8}\rho U^2\sqrt{\frac{8RT}{\pi}}\frac{1}{d/2\lambda+1} \tag{21}$$

This is essentially the empirical viscous dissipation model proposed by Ju for Couette flow[6], which gives results that agree well with his DSMC results. Ju explained the difference between Eqs. (14) and (19) as a result of the less frequent intermolecular collisions than collisions between fluid molecules and the plates. As shown in our derivation, Eqs. (14) and (19), however, correspond to the heat transfer in different cases, and their difference is caused by the flow velocity slip at the boundary plates. Thus, Zhang and Bogy's slip-flow-based result[4] and Ju's semi-analytical result[6] for the Couette-flow caused viscous dissipation refer to different heat fluxes.

Using a more accurate expression of the shear stress in Couette flow, valid for the entire Knudsen number regime[17], we get the following analytical formula for the viscous dissipation due to the Couette flow at the upper boundary plate, i.e. the bearing surface, as,

$$q_y\big|_{upper} = \rho \frac{u_0^2}{4} \sqrt{\frac{2RT}{\pi}} \frac{0.5296\mathrm{Kn}^2 + 1.2058\mathrm{Kn}}{0.5296\mathrm{Kn}^2 + 1.6276\mathrm{Kn} + 0.6029} \tag{22}$$

ii) Viscous dissipation due to the Poiseuille flow

Viscous dissipation in plane Poiseuille flow, as shown by Fig. 6(a), is analyzed in a similar manner as for the Couette flow. We assume that the gas pressure p satisfies $h(\partial p/\partial x)/p_0 << 1$, where p_0 is the ambient gas pressure, and the pressure-driven flow is subsonic. Since the gas film thickness is small, the velocity u_y is also negligible. Here, we again consider two cases. We first take a control volume of the flow without boundary plates, as shown by Fig. 6(b). The steady state linear momentum conservation of the flow in the control volume is,

$$\int_0^h \left(\frac{\partial(\rho u_x u_x)}{\partial x} - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) dy = 0 . \tag{23}$$

Notice that for subsonic flows $\frac{\rho u_x u_x}{p} = (\frac{u_x}{\sqrt{RT}})^2 << 1$. Hence $\rho u_x u_x - p$ can be approximated by – p. The symmetry of Poiseuille flow with respect to the center line produces $\tau_{xy}|_{y=0} = -\tau_{xy}|_{y=h}$. Finally Eq. (23) gives,

$$\tau_{xy} = \frac{h}{2}\frac{\partial p}{\partial x}. \tag{24}$$

This agrees with the result obtained by Bahukudumbi and Beskok[18], but here fewer assumptions are needed.

The integral form of the steady state energy conservation at the control volume in Fig. 6(b) is,

$$\int_0^h \frac{\partial}{\partial x}[\rho u_x(\eta + \frac{u_x^2 + u_y^2}{2})]dy = \int_0^h \frac{\partial}{\partial x}(-q_x)dy + \int_0^h \frac{\partial}{\partial y}(\tau_{xy}u_x - q_y)dy. \tag{25}$$

Notice that $\eta = e + p/\rho$, where e is the gas internal energy per unit mass, and that $\rho u_x u_x/p << 1$. The left hand side of Eq. (25) can be approximated by $\int_0^h \partial(\rho u_x \eta)/\partial x dy$. The symmetry of Poiseuille flow with respect to its center line gives $q_y|_{y=0} = -q_y|_{y=h}$ and $u_x|_{y=0} = u_x|_{y=h}$. So finally Eq. (25) can be transformed to,

$$q_y\Big|_{y=h} = -\frac{1}{2}\int_0^h \frac{\partial}{\partial x}(\rho u_x \eta + q_x)dy + \frac{h}{2}\frac{\partial p}{\partial x}u_x\Big|_{y=h}. \tag{26}$$

This is the heat flux in the gas flow at $y = h$ contributed from viscous dissipation.

Now we consider the second case and take a control volume that includes the thin boundary plates, as shown by Fig. 6(c). The total heat flux in this case is

$$q_y\Big|_{upper} = -\frac{1}{2}\int_0^h \frac{\partial}{\partial x}(\rho u_x \eta + q_x)dy, \tag{27}$$

since the boundary plates remain stationary.

This heat flux contributed by viscous dissipation reduces to zero if two assumptions of the flow are imposed. If the flow is still assumed to be near isothermal and the gas enthalpy is $\eta = C_pRT$, where C_p denotes the gas specific heat at constant pressure and is constant, we can show that $\partial(\rho u_x \eta)/\partial x = C_p RT \partial(\rho u_x)/\partial x = 0$, since the steady state mass conservation equation gives the relation $\partial(\rho u_x)/\partial x = 0$. If it is also assumed that the derivative $\partial(q_x)/\partial x$ vanishes, finally we have $q_y\big|_{upper} = 0$. Under the above two assumptions, Eq. (26) reduces to $q_y\big|_{y=h} = (h/2)(\partial p/\partial x)u_x\big|_{y=h}$, which is what Shen and Chen obtained. In a slip flow, $u_x\big|_{y=h} = -(1/2\mu)(\partial p/\partial x)\lambda h(2-\alpha)/\alpha$, and Eq. (26) in the flow at $y = h$ is written as $q_y\big|_{y=h} = -(\lambda h^2/4\mu)(\partial p/\partial x)(2-\alpha)/\alpha$. This is the same as the expression obtained by Chen *et al.* for slip flows[5]. Following our derivation, Eq. (26), however, only accounts for the part of the total viscous dissipation corresponding to the Poiseuille flow. It should be emphasized that the *total* heat flux at the upper boundary plate is $q_y\big|_{upper} = 0$, if the flow is still isothermal and the heat flux in x-direction does not change.

IV. CONCLUSIONS

A phenomenological heat transfer model for the molecular gas lubrication system in hard disk drives is proposed. The conduction heat flux due to the temperature difference of the slider and disk is obtained using the temperature jump theory and the modified mean free path where the effects of the presence of the two boundaries are included. The obtained conduction heat flux agrees better with the results from the linearized Boltzmann equation than previous models based on the classical mean free path. The linearized Boltzmann equation has been shown to be incapable of calculating the heat flux contributed by the viscous dissipation in Couette and Poiseuille flows. Instead, here the viscous dissipation in the gas lubrication film is analyzed through the energy conservation equation. The viscous heat flux at the gas film boundary is different from that at the corresponding boundary plate in either Couette flow or Poiseuille flow, due to the gas slip at the boundary. Expressions for the heat flux contributed from viscous dissipation in Couette flow and Poiseuille flow are obtained separately.

ACKNOWLEDGEMENT

This research was supported by the Information Storage Industry Consortium (INSIC) and the Computer Mechanics Laboratory (CML) at the University of California at Berkeley.

[1] H. Tian, C. Y. Cheung, and P. K. Wang, *IEEE Trans. Magn.* **33**, 3130 (1997).

[2] D. W. Meyer, P. E. Kupinski, and J. C. Liu, “Slider with temperature responsive transducer positioning,” U. S. Pat. No. 5,991,113 (1999).

[3] S. Song, K. Stoev, D. Fang, W. Gan, V. Rudman, and J. Wang, *J. Appl. Phys.* **97**, 10P303 (2005).

[4] S. Zhang and D. B. Bogy, *Int. J. Heat Mass Transfer* **42**, 1791 (1997).

[5] L. Chen, D. B. Bogy, and B. Strom, *IEEE Trans. Magn.* **36**, 2486 (2000).

[6] Y. S. Ju, *J. Heat Transfer* **122**, 817 (2000).

[7] S. Shen and G. Chen, *J. Appl. Phys.* **103**, 054304 (2008).

[8] Y. Peng, X. Lu, and J. Luo, *ASME J. Tribol.* **126**, 347 (2004).

[9] P. Bassanini, C. Cercignani, and C. D. Pagani, *Int. J. Heat Mass Transfer* **11**, 1359 (1968).

[10] W. D. Zhou, B. Liu, S. K. Yu, W. Hua, and C. H. Wong, *Appl. Phys. letters* **92**, 043109

(2008).

[11] K. Koura and H. Matsumoto, *Phys. Fluids* **A 4**, 1083 (1992).

[12] Y. Sone, *Molecular Gas Dynamics: Theory, Techniques, and Applications* (Birkhäuser, Massachusetts, 2007)

[13] Y. Sone, *Theory and Numerical Analysis of the Boltzmann Equation* (Birkhäuser, Massachusetts, 2002).

[14] S. Fukui and R. Kaneko, *ASME J. Tribol.* **110**, 253 (1988).

[15] W. G. Vincenti and C. H. Kruger, *Introduction to Physical Gas Dynamics* (Krieger, Florida, 1965).

[16] C. Y. Liu and L. Lees, In *Rarefied Gas Dynamics*, edited by L. Tablet (Academic Press, New York-London, 1961).

[17] P. Bahukudumbi, J. H. Park, and A. Beskok, *J. Microscale Thermophys. Eng.* **7**, 291 (2003).

[18] P. Bahukudumbi and A. Beskok, *J. Micromech. Microeng*. **13**, 873 (2003).

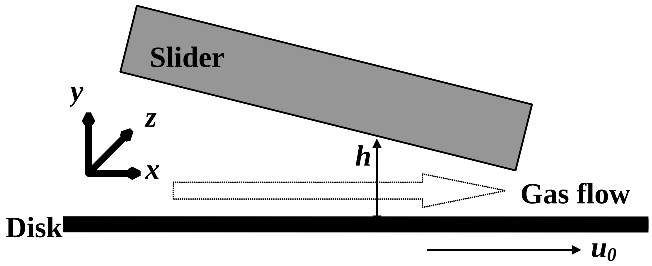
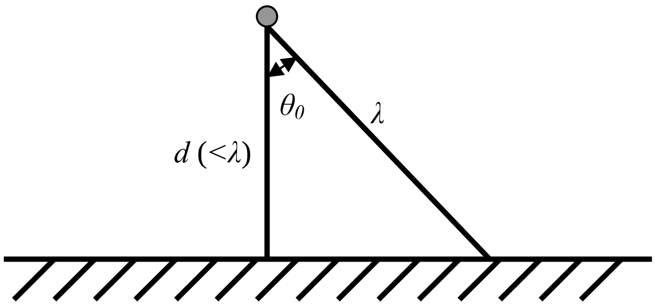
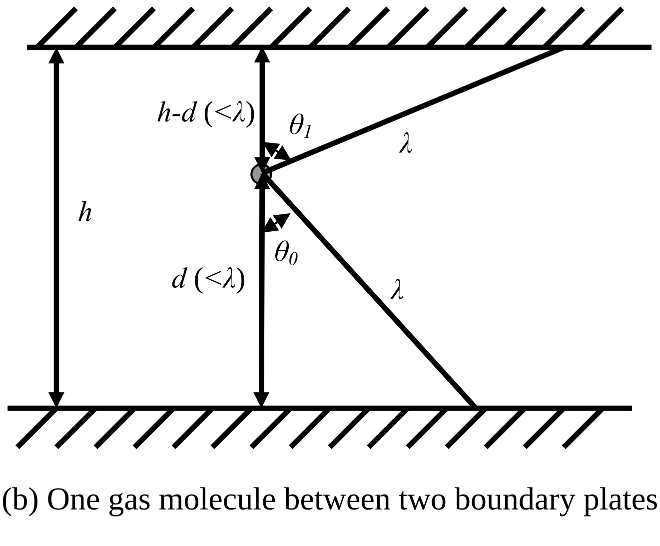


FIG. 1. Gas lubrication film in the head disk interface between the slider and the disk



(a) One gas molecule and one boundary plate



(b) One gas molecule between two boundary plates

FIG. 2. One gas molecule moves close to a plate (a) or between two plates (b)

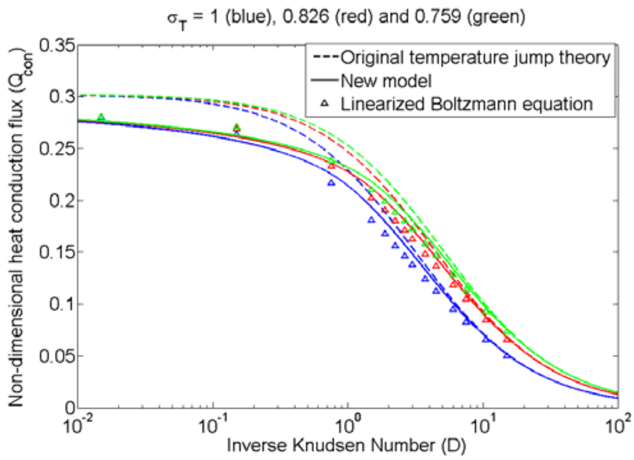


FIG. 3. Comparison of the non-dimensional heat conduction flux (Q_{con}) between two parallel plates obtained from the new model with the original temperature jump theory model[4] and the linearized Boltzmann equation[9].

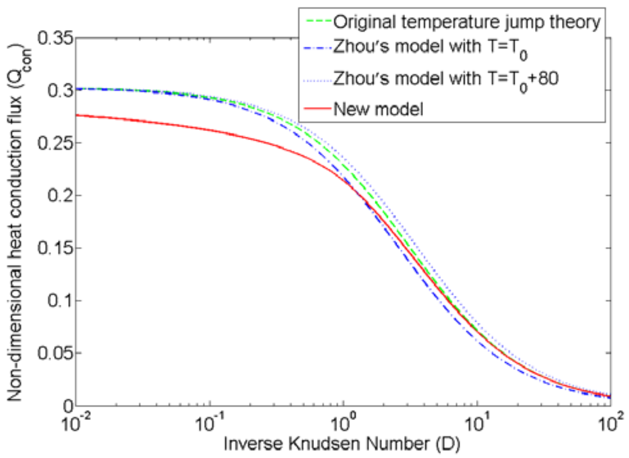
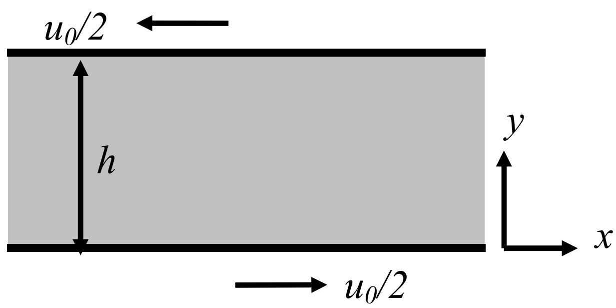
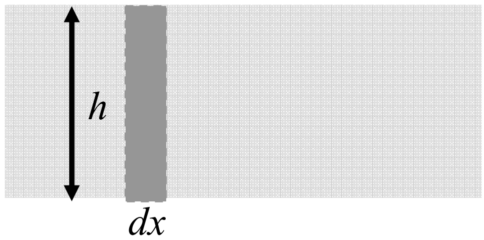


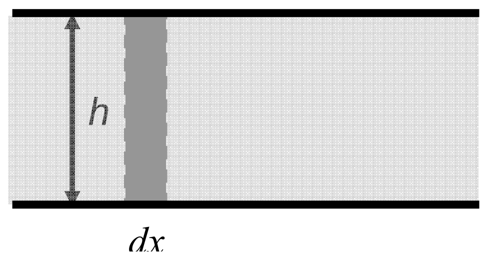
FIG. 4. Comparison of the non-dimensional heat conduction flux (Q_{con}) between two parallel plates obtained from the new model with the original temperature jump theory model[4] and Zhou's model[10].



(a) Plane Couette flow

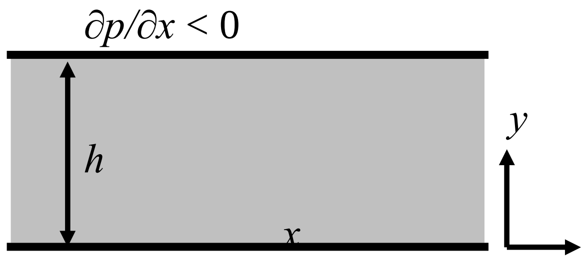


(b) Control volume of gas flow

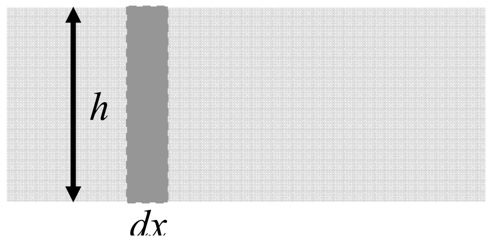


(c) Control volume including the upper and lower boundary plates

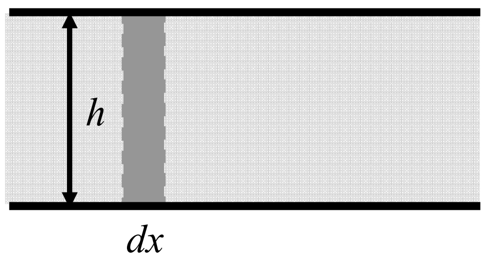
FIG. 5. Couette flow between two plates of the same temperature (a), a control volume of gas flow (b) and a control volume of gas flow with upper and lower boundaries (c)



(a) Plane Poiseuille flow



(b) Control volume of gas flow



(c) Control volume including the upper and lower boundary plates

FIG. 6. Poiseuille flow between two plates of the same temperature (a), a control volume of gas flow (b) and a control volume of gas flow with upper and lower boundaries (c)