A Novel Design for Short Seeking Control by Optimal Finite-step Feed-forward Control

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ABSTRACT

Two-degree-of-freedom (2DOF) control is a popular method for short seeking control of hard disk drive, which requires design and implementation of reference trajectory and feed-forward controllers. In this report, a new feed-forward control design method based on optimization technique is proposed for short seeking control of HDDs. The feedforward control input is of finite time steps and is calculated to minimize a cost function defined by position error, control efforts, and jerk. The proposed method achieves smooth and fast short seeking by the optimal finite-step feedforward control without using reference trajectories, which also implies that real time computation and memories can be saved in its implementation. The design example confirmed that an excellent short-seeking performance is achieved with a small amount of real time computation using the proposed method.

Keywords: Short seeking, Feed-forward control, Two-degree-of-freedom (2DOF) control, Hard disk drives (HDDs)

1 Introduction

The hard disk drive (HDD) industry continues to strive for increased storage densities and reduced data access times. This demands performance improvements of the head positioning system in terms of fast movement and precise positioning. For this reason, a significant amount of research efforts has been devoted to apply advanced control methods to HDDs.

In general, the servo systems of HDDs have two major tasks: one is track following and the other is track seeking. Track following maintains the read/write head on the center of a specified track as precisely as possible. In track following, it is important to regulate the tracking error against various disturbances. Track seeking moves the head from the present track to a target track as quickly as possible. In track seeking, near time-optimal control is generally used, especially for long span seeking: e.g. proximate time optimal servomechanism (PTOS) [1]-[3]. The read/write head follows such an optimal trajectory to move toward the target track. As the head comes close to the target track, the controller switches from the seek mode to the settling mode, and finally to the track-following mode. However, for short-span seeking, it is more effective to use unified control schemes, in which track seeking and track following control can be simultaneously applied. A popular scheme for short seeking is based on a two-degree-of-freedom (2DOF) servo structure

[4]-[7]. Typically, the track following controller is used in the 2DOF servomechanism as the feedback controller, and the feedforward controller is tuned according to the closed loop dynamics along with the design of the reference trajectory. The zero phase error tracking (ZPET) [8] technique can be utilized to design the feedforward controller, and minimal jerk trajectory [9]-[10] is a popular reference generation method for short seeking/settling control of disk drives.

In this report, we proposed a novel feed-forward control design method based on optimization technique for short seeking control of HDDs. In this method, the feedforward control input is of finite time steps and is calculated to minimize a cost function. By incorporating the tracking error, smoothness of head position, control efforts and jerk of the feed-forward control input in the cost function, optimal finite-step feed-forward control can achieve desired short seeking without using any reference trajectory. As a consequence, real time computation and memories can be saved in its implementation.

The remainder of this report is organized as follows. In Section 2, the proposed control structure is presented. In Section 3, the formulation of the optimization problem and the design of algorithm are discussed. Section 4 shows a design example and Section 5 concludes this report.

2 Proposed Control Structure and System Description

Figure 1 shows a schematic diagram of the proposed short seeking control system by optimal finite-step feed-forward control for HDDs. *P* represents the plant and *C* is the track following controller. *f*(*k*) is the feed-forward control signal.

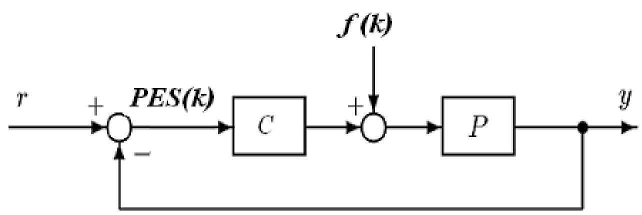


Fig. 1 Schematic diagram of short-seek control
with optimal finite-step feedforward control for HDDs

The proposed system has the following features:

(1) The control structure during seeking is a two-degree-of-freedom (2DOF) structure.

(2) Reference input is set to be a step signal for short seeking, i.e., *r(k)=r* for $k \geq 0$, where *r* is the seeking distance.

(3) Feed-forward control signal *f*(*k*), which consists of a finite number of pulses, i.e., $f(k) = 0$ for $k > n$, is added to the control input to shape the transient response in short seeking.

In our design, the feedforward control signal *f*(*k*) can be represented as the impulse response of a finite impulse response (FIR) filter F(z):

$$F(z) = f(0) + f(1)z^{-1} + ... + f(n)z^{-n} \tag{1}$$

where the order of the FIR filter equals to the number of pulses in the feedforward control signal $f(k)$, and each coefficient of the FIR filter corresponds to the magnitude of each pulse in $f(k)$.

From Fig.1 and E.q. (1), the position error signal PES(k) can be represented as follows.

$$PES(k) = \frac{1}{1+C(z)P(z)} \cdot r \cdot 1(k) - \frac{P(z)}{1+C(z)P(z)} \cdot F(z) \cdot \delta(k) \tag{2}$$

where $1(k)$ is a unit step function and $\delta(k)$ is a unit impulse function.

Notice that the feedforward scheme improves the transient response without changing the closed-loop characteristics such as stability and sensitivity. A methodology on how to find the optimal feedforward control signal, $f(k)$ for $0 \le k \le n$, is presented in the next section.

3 Problem Formulation and Algorithm Design

3.1 Problem Formulation

Define the feedforward control signal vector of length n+1 as

$$f = [f(0), f(1), ..., f(n)] \tag{3}$$

The optimization-based design is a search for f to minimize a cost function.

To obtain good performance in short seeking, we define the cost function based on the squared summation of the position error, the successive difference, i.e.smoothness, of the output, the feedforward control input and the successive difference of the feed-forward control input for enhanced performance as follows:

$$J = \sum_{k=1}^{\infty}\{PES^2(k) + q_1 \cdot [PES(k+1) - PES(k)]^2\} + \sum_{k=0}^{n}\{q_2 \cdot f^2(k) + q_3[f(k+1) - f(k)]^2\} \qquad (4)$$

where $q_1, q_2,$ and q_3 are weighting factors to characterize the performance requirements. Thus, short seeking by finite-step feed-forward control can be formulated as the following optimization problem.

<u>Problem 1</u>: Consider the servo control system depicted in Fig. 1 or the system descried by E.q. (2). Find the feedforward control signal vector,

$$f = [f(0),\ f(1), \ldots,\ f(n)] \qquad (3)$$

that minimizes the cost function *J:*

$$J = \sum_{k=1}^{\infty}\{PES^2(k) + q_1 \cdot [PES(k+1) - PES(k)]^2\} + \sum_{k=0}^{n}\{q_2 \cdot f^2(k) + q_3[f(k+1) - f(k)]^2\} \qquad (4)$$

where $q_1 > 0, q_2 > 0, q_3 > 0$ are weighting factors.

3.2 Representation of Position Error Signal via State Space

Since
$$1(t) = \frac{z}{z-1} \cdot \delta(t) \tag{5}$$

PES(k) in (2) can be represented as

$$PES(k) = [\frac{r}{1+C(z)P(z)} \cdot \frac{z}{z-1} - \frac{P(z)F(z)}{1+C(z)P(z)}] \cdot \delta(k) \tag{6}$$

From (6) and (1),

$$PES(k) = (1 - \frac{C(z)P(z)}{1+C(z)P(z)})[r \cdot (1+\frac{1}{z-1}) - \frac{f(0)z^n + f(1)z^{n-1} + \ldots f(n)}{z^n} P(z)] \cdot \delta(k)$$

After some manipulations, we obtain that

$$PES(k) = (r + \frac{N(z)}{D(z)}) \cdot \delta(k) \tag{7}$$

where (i) the order of polynomial N(z) is strictly less than that of D(z);

(ii) D(z) is determined by C(z) and P(z), and is independent of F(z);

(iii) N(z) is dependent on *f*, C(z) and P(z).

From above, PES(k) can be represented in state space as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + B\delta(k) \\ PES(k) &= Cx(k) + r\delta(k) \end{aligned} \tag{8}$$

where (i) *A* is determined by C(z) and P(z), and is independent of F(z);

(ii) *B*= [0 … 0 1]';

(iii) $C = c_0 + f \cdot D$ is linearly dependent on *f*. (9)

After PES(k) is represented in state space form as Eq.(8), we can solve the optimization

Problem 1, and obtain the solution as follows.

3.3 Optimal Finite-step Feedforward Control

Theorem 1: Consider Problem 1 and Eq. (8), The optimal finite-step feedforward control signal that minimizes the cost function J is:

$$f^* = -c_0 D\overline{W}[D\overline{W}D^T + q_2 I + q_3 D_f{}^T D_f]^{-1} \tag{10}$$

where

$$\overline{W} =: (1+2q_1)W - q_1(AW + WA^T + BB^T) \tag{11}$$

W is obtained by solving the Lyapunov equation:

$$AWA^T - W + BB^T = 0 \tag{12}$$

and

$$D_f = \begin{bmatrix} -1 & 1 & & & & \\ & -1 & 1 & & & \\ \ldots & & & & & \\ 0 & 0 & \ldots & -1 & 1 \\ 0 & 0 & & 0 & -1 \end{bmatrix}_{(n+1)\times(n+1)} \tag{13}$$

Proof:

First, we consider the first term in the cost function.

$$
\begin{aligned}
J_0 &= \sum_{k=1}^{\infty} PES^2(k) = \sum_{k=1}^{\infty} Cx(k)x(k)^T C^T = \sum_{i=1}^{\infty} CA^{i-1}BB^T (A^T)^{i-1}C^T \\
&= C(\sum_{i=1}^{\infty} A^{i-1}BB^T (A^T)^{i-1})C^T \\
&=: CWC^T ,
\end{aligned}
\tag{14}
$$

where $W =: \sum_{i=1}^{\infty} A^{i-1}BB^T (A^T)^{i-1}$ satisfies the Lyapunov equation:

$$AWA^T - W + BB^T = 0$$

which has a unique positive semi-definite solution if *A* is stable.

Now we consider the first two terms in the cost function.

$$J_1 = \sum_{k=1}^{\infty} \{PES^2(k) + q_1[PES(k+1) - PES(k)]^2\} \tag{15}$$

Using (14) and (8), (15) can be transformed into

$$J_1 = C\overline{W}C^T \tag{16}$$

where

$$\overline{W} =: (1+2q_1)W - q_1(AW + WA^T + BB^T) \tag{17}$$

To represent the last two terms of the cost function in a convenient form, we define

$$f_d = \begin{bmatrix} f(1) - f(0) \\ f(2) - f(1) \\ \dots \\ f(n) - f(n-1) \\ f(n+1) - f(n) \end{bmatrix} \tag{18}$$

and note

$$f_d = \begin{bmatrix} f(1) - f(0) \\ f(2) - f(1) \\ \dots \\ f(n) - f(n-1) \\ f(n+1) - f(n) \end{bmatrix} = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ \dots & & & & \\ 0 & 0 & \dots & -1 & 1 \\ 0 & 0 & & 0 & -1 \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \dots \\ f(n) \end{bmatrix} = D_f f^T \tag{19}$$

Then,

$$
\begin{aligned}
J_2 &= \sum_{k=0}^{n} \{q_2 f^2(k) + q_3[f(k+1) - f(k)]^2\} \\
&= q_2 ff^T + q_3 f_d^{\ T} f_d \\
&= q_2 ff^T + q_3 fD_f^{\ T} D_f f^T
\end{aligned} \tag{20}
$$

Combining (15) and (20), and using (9), we obtain

$$
\begin{aligned}
J &= \sum_{k=1}^{\infty} \{PES^2(k) + q_1[PES(k+1) - PES(k)]^2\} + \sum_{k=0}^{n} \{q_2 f^2(k) + q_3[f(k+1) - f(k)]^2\} \\
&= J_1 + J_2 \\
&= C\overline{W}C^T + q_2 ff^T + q_3 fD_f^{\ T} D_f f^T \\
&= (c_0 + fD)\overline{W}(c_0 + fD)^T + q_2 ff^T + q_3 fD_f^{\ T} D_f f^T \\
&= f[D\overline{W}D^T + q_2 I + q_3 D_f^{\ T} D_f] f^T + 2c_0 D\overline{W} f^T + c_0 \overline{W} c_0^{\ T}
\end{aligned} \tag{21}
$$

To minimize the above function with respect to *f*, we can set

$$
\frac{\partial J}{\partial f} = 2f[D\overline{W}D^T + q_2 I + q_3 D_f^{\ T} D_f] + 2c_0 D\overline{W} = 0 \tag{22}
$$

Therefore, the optimal coefficient vector f^*can be obtained as:

$$
f^* = -c_0 D\overline{W}[D\overline{W}D^T + q_2 I + q_3 D_f^{\ T} D_f]^{-1} \tag{23}
$$

Note that the optimal feedforward control signal sequence only needs to be calculated once during the design process using (10), and it may be stored in the memory as a look-up table. In implementation, the optimal feedforward control signal can be obtained from the look-up table, which implies that the optimization procedure does not increase the amountof real time computation.

4 Design Example

In this section, the proposed method is applied to a HDD and evaluated by simulations. We obtain the plant model from a real HDD, which can be well approximated by a 29th-order model. Fig. 2 shows the frequency responses of the plant model. The solid line in Fig. 2 is frequency response of the 29th –order plant model, and dashed line is frequency response a 4th-order approximate model. We will design the control algorithms based on the 4th-order plant model, and evaluate the performance based on both the 4th-order plant model and the 29th-order plant model as well as a more realistic plant which captures more characteristics of the HDD than the 29th-order plant, such as nonlinearities, bias, etc.

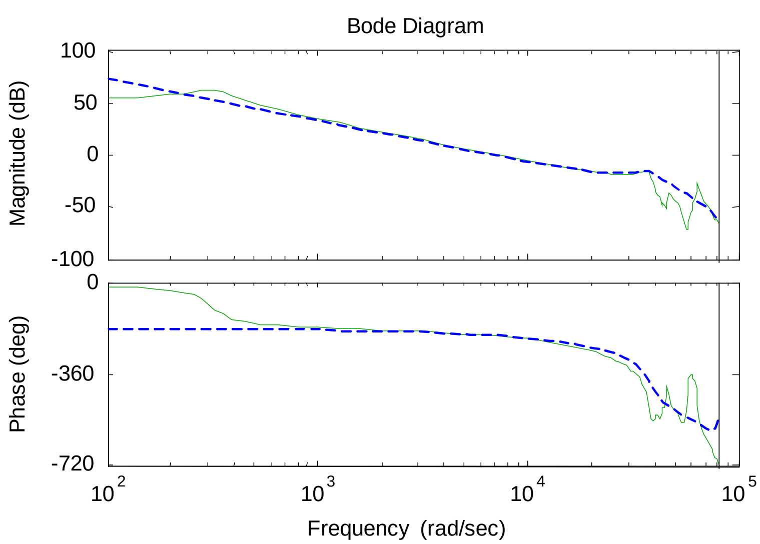


Fig. 2 Frequency responses of the plant model

Fig. 3 shows one-track seeking using the proposed method. The solid line shows the response of the proposed method and the dashed line shows the response of the output without optimal finite-step feed-forward control. From this figure, we can see that a fast and smooth response in short seeking can be achieved by optimal finite-step feed-forward control. Fig. 4 shows the optimal 5- step feed-forward control signal applied.

Figure 5 indicates how the transient characteristics may be shaped by different weighting factors q_1 in the cost function. Here we set q_2, q_3 to be 0. The upper figure shows the read/write head position and the lower figure shows the feed-forward control input. We can observe from Fig. 5 that smaller weighting factors q_1 ($q_1=0$) give faster responses but may lead to overshoot of the head position, and larger weighting factors q_1 (e.g. $q_1=1.45$), which imply that the fluctuation of position error is penalized more relative to other terms in the cost function, give smoother but slower responses. As a result, we can tune the weighting factor q_1 to achieve desired step-responses accounting for both the speed and overshoot. It is worth noting that if the feed-forward control signal is not taken into account in the cost function (i.e. q_2 and q_3 are set to 0), the feed-forward control input fluctuates a lot, which is not desirable. Therefore, it is important that we include the feed-forward control input in the cost function.

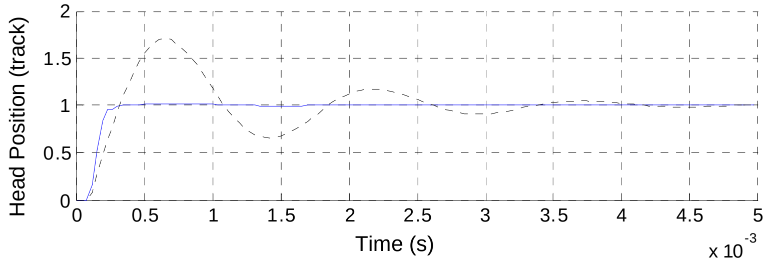


Fig. 3 1-track seeking with/without finite-step feed-forward control

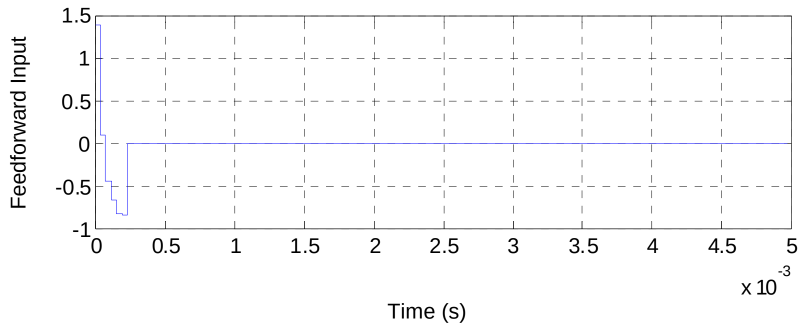
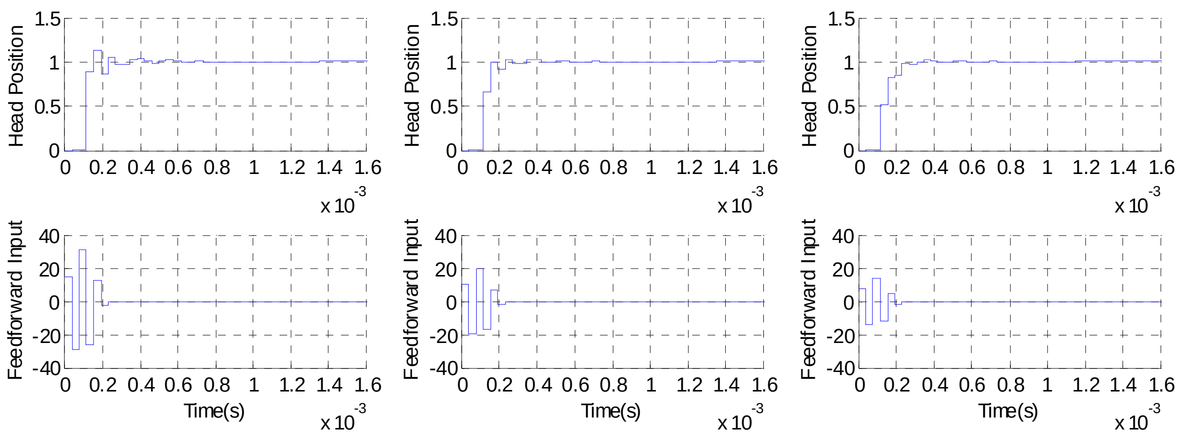


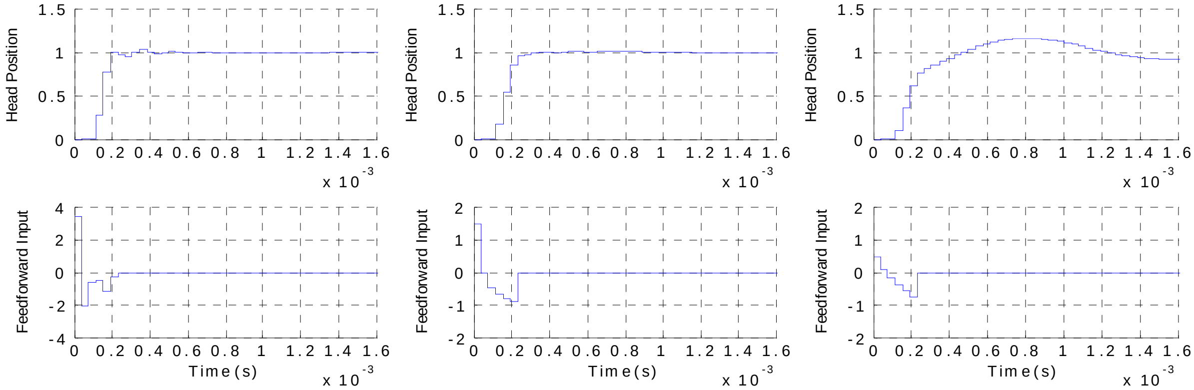
Fig. 4 Feedforward control signal



q_1=0, q_1=0.5, q_1=1.45

Fig. 5 1-track seeking for different weighting factor q_1

Upper figure: head position; lower figure: feedforward control input



q_2=0.008 q_2=0.08, q_2=0.8

Fig. 6 1-track seeking for different weighting factor q_2

Upper figure: head position; lower figure: feedforward control input

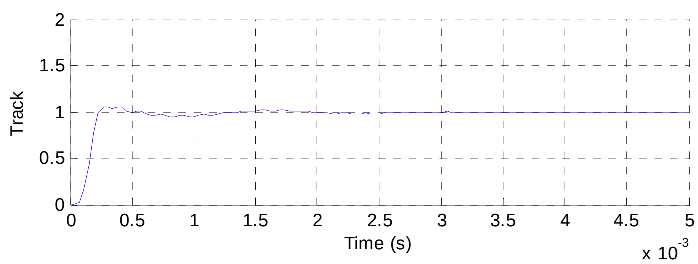


Fig. 7 1-track seeking using the proposed method;

the plant is represented by a 29-th order model

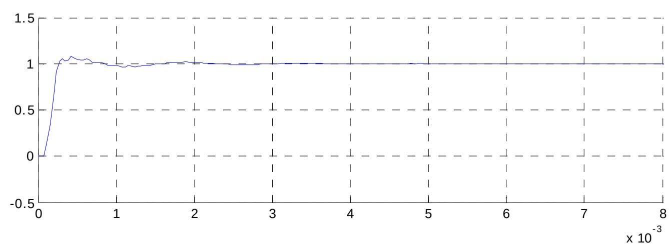


Fig. 8 1-track seeking using the proposed method;

the plant is represented by a realistic model which includes nolinearities and bias

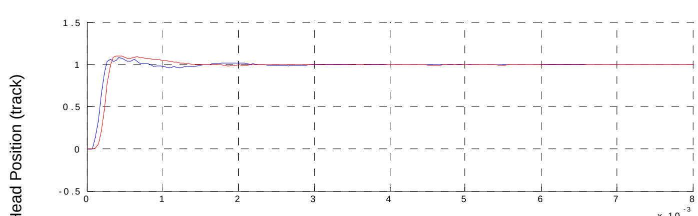


Fig. 9 1-track seeking using the proposed method and using the standard 2DOF design;

the plant is represented by a realistic model which includes nonlinearities and bias

Figure 6 shows how the transient characteristics can be shaped by different weighting factors q_2 in the cost function. Here we have set q_1 to 0.5. The upper figure shows the read/write head position and the lower figure shows the feed-forward control input. We can observe from Fig. 6 that a small weighting factor q_2 ($q_2=.008$), which allows for a large control input, gives a fast response but may lead to an overshoot of the head position, and a large weighting factor q_2 ($q_2=0.8$) gives a smooth feed-forward control input but a much slower response. Thus, we can tune the weighting factor q_2 to achieve desired step-responses taking into account both the smoothness of the feedforward control input and the settling time of the response. It is also noted that since both the magnitude and the successive difference of the feed-forward control signal are taken into account in the cost function in this simulation, the feed-forward control inputs become much smaller and smoother compared with those in Fig. 5..

We also evaluate the short seeking performance using the 29th-order plant and a more realistic plant which captures more characteristics of the HDD than the 29th-order plant, such as nonlinearities and bias. Figure 7 shows 1-track seeking using the proposed method for the 29-th order plant model. Fig. 8 shows 1-track seeking using the proposed method for the more realistic plant model. In both cases here, we applied the same 5-step feed-forward control input shown in Fig. 4. We can see a fast and smooth short seeking performance is retained. Figure 9 and *Table I* compare the 1-track seeking performance using the proposed method and using the standard 2DOF design (which requires a shaped reference trajectory and a feedforward controller) for the more realistic plant model. It is observed that the proposed method can reduce the overshoot by 19.4% and the 10%-settling time by 20.7% with a lesser amount of real time computation load, compared to the standard 2DOF design.

TABLE I. SHORT-SEEKING PERFORMANCE USING THE STANDARD 2DOF DESIGN AND THE PROPOSED METHOD

Performance Spec	Standard 2DOF method	Proposed method
5% Settling Time (ms)	0.95	0.64
10% Settling Time (ms)	0.29	0.23
OverShoot (Track)	9.8%	7.9%

5 Conclusions

This report has proposed a novel method of short seeking control by optimal finite-step feedforward control. This method, which uses the track following controller as feedback controller, introduces a finite-step feed-forward control signal to improve the transient response in short seeking. The short seeking control problem was formulated as an optimization problem by finding the finite-step feed-forward control signal to minimize a cost function. By incorporating the tracking error, the smoothness of head position, both the magnitude and the successive difference of the feed-forward control input in the cost function, optimal finite-step feed-forward control can achieve smooth and fast short seeking without using reference trajectories, which also implies a lesser amount of real time computation and memories in its implementation.

The design example confirmed that an excellent short-seeking performance can be achieved with a small amount of real time computation using the proposed method.

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