# A NOVEL DESIGN OF SHORT-SEEKING CONTROL FOR HARD DISK DRIVES

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## ABSTRACT

**This report discusses short seeking control for a hard disk drive with voice coil motor (VCM). A novel design method is proposed based on the track following control and initial value adjustment (IVA). Initial value adjustment, which adjusts the initial values of the track-following controllers for short seeking, makes the use of the feed-forward controller and reference trajectory unnecessary. The proposed design also takes into consideration the compromise between speed and overshoot in short seeking when seeking distance becomes longer by using a two-stage IVA scheme: (1) In the first stage (when the read/write head is far away from the target track), the integral action of the track following controller is inactivated and IVA is designed to yield a high-speed seeking; (2) In the second stage (when the head approaches the target track), the integral action is switched on, and IVA is designed to produce a fast and smooth response without overshoot. A design example verifies the effectiveness of the proposed method.**

**Keywords:** **Short seeking, Initial value adjustment (IVA), Hard disk drives (HDDs)**

# 1. INTRODUCTION

The hard disk drive (HDD) industry continues to strive for increased storage densities and reduced data access times. This demands performance improvements of the head positioning system in terms of fast movement and precise positioning. For this reason, many research efforts have been devoted to apply advanced control methods to HDDs.

In general, the servo systems of HDDs have two major tasks: one is track following and the other is track seeking. Track following maintains the read/write head on the center of a specified track as precisely as possible. In track following it is important to regulate tracking error against various disturbances. Track seeking moves the head from present track to a target track as quickly as possible. In track seeking, near time-optimal control is generally used, especially for long span seeking: e.g. proximate time optimal servomechanism (PTOS) [1]-[3]. The read/write head follows such an optimal trajectory to move toward the target track, then switches to settling mode, and finally into track-following mode. However, for short-span seeking, it is more effective to use unified control schemes, in which the track seeking and track following control can be simultaneously applied. A popular scheme for short seeking is based on a two-degree-of-freedom (TDOF) servo structure [4]-[7]. However, the TDOF control algorithm is much more complicated due to the feed-forward controller and reference generation, which also implies more real time computation and memories are required in its implementation. To save real time computation, Hirata and

Tomizuka avoided the use of TDOF structure in short seeking by utilizing multi-rate control and initial value compensation [8].

In this report, we propose a novel method of short seeking control based on track following control and initial value adjustment (IVA). This method, which uses the same controller as in track following, tunes the initial value of track-following controller for short seeking. By tuning the initial value of the feedback controller, the desired transient characteristics in short-seeking can be obtained without the use of feed-forward controller and reference trajectory, so that no real time computation for feed-forward control is needed. On the other hand, almost all track following controllers include an integral action for improved steady state error, which leads to the compromise between speed and overshoot in short seeking when seeking distance becomes longer. As such, a two-stage IVA scheme is presented: (1) The integral action of the track following controller is inactivated in the first stage (when the read/write head is away from target track), and IVA is designed to yield a high-speed seeking; (2) The integral action is switched on when the head approaches the target track in the second stage, and IVA is designed to produce a fast and smooth response without overshoot.

The remainder of this report is organized as follows. Section *2* describes the structure of a HDD and the track-following controller design. Section *3* presents the design of IVA for short seeking control; and in Section *4*, the design of two-stage IVA is discussed in detail along with a design example. Conclusions are given in Section *5*.

## 2. HARD DISK DRIVE SYSTEM AND TRACK FOLLOWING CONTROLLER

A typical hard disk drive (HDD) is shown in Fig. 1 (a). It has read/write heads, a voice coil motor (VCM) and magnetic disks which are rotated by a spindle motor. An actual HDD was modeled through frequency response test and Fig. 1 (b) shows the experimental setup which includes the HDD, a digital signal processor (DSP), and a laser Doppler vibrometer for measuring the position of the read/write head. The frequency responses of the plant model are shown in Fig.2. Notice that the plant exhibits second order characteristics with three resonance modes at high frequencies. The gain plot is flat at low frequencies due to pivot friction.

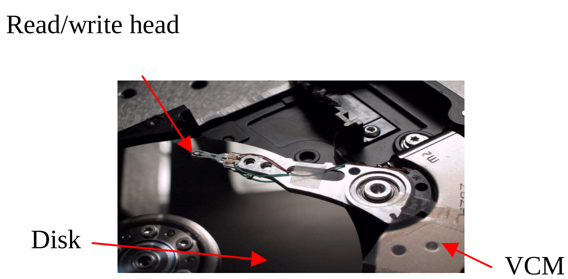



Fig. 1 (a) Dual-actuator Hard Disk Drive

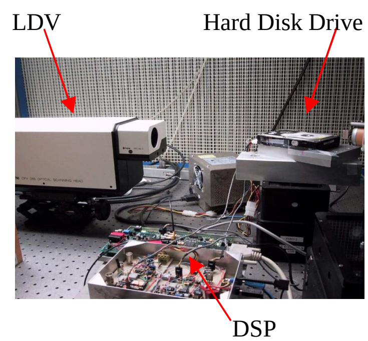



Fig. 1 (b) Experimental Setup

The servo structure for track following is as depicted in Fig.3, where $P_{VCM}$ and C represent the plant model and track following controller, respectively. The control input $u$ is a voltage for VCM and the measurement output $y$ is the head position. Suppose a track following controller has already been designed to meet performance specifications for the plant, which include stability of the closed loop system and steady state error.

As an example, we consider a third order lead-lag compensator described by

$$C(z) = 4.0681\frac{(z - 0.9057)(z - 0.9756)(z - 0.9905)}{(z - 0.8664)(z - 0.9577)(z - 1)} \tag{1}$$

This controller possess a pole at z=1, which corresponds to an integral action for zero steady state error for constant disturbances.

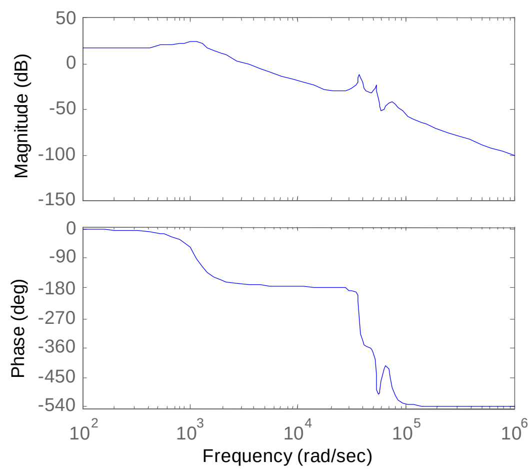



Fig. 2 Frequency responses of the plant

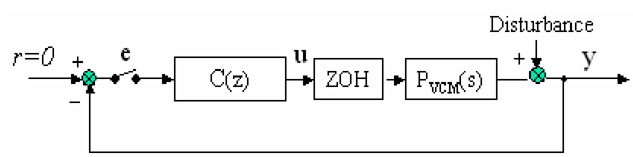



Fig 3 Tracking following control system for HDDs

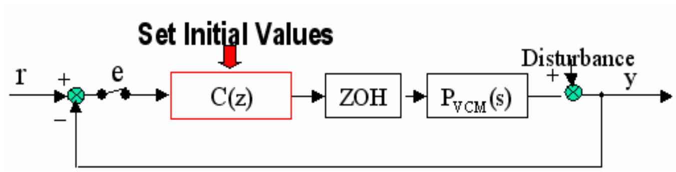



Fig. 4 Short seeking with IVA for HDDs

# 3. SHORT-SEEKING WITH INITIAL VALUE ADJUSTMENT

We will first explain short-seeking control by IVA when seeking distance is very short in this section, and discuss short-seeking by two-stage IVA when seeking distance becomes longer in the next section.

## 3.1 Overall Control Structure for Short Seeking by IVA

Fig. 4 shows a schematic diagram of short seeking by IVA of the track-following controller. $P_{VCM}$ represents the dynamics of the plant and $C(z)$ is a track following controller. The proposed scheme has the following features:

(1) The overall structure during seeking is a one-degree-of-freedom structure, and the feedback controller $C(z)$ is the same as the track following controller.

(2) Reference input is set to be a step signal, i.e., $r(k)=r$ for $k \geq 0$, where $r$ is the seeking distance

(3) Nonzero initial value is set to the controller for short seeking.

Notice that the adjustment of the initial values of the controller improves the transient response without changing closed-loop characteristics such as stability and sensitivity. Since we already have the track following controller, finding the initial value of the controller becomes the main issue. In the following section *3.2*, the closed loop system depicted in Fig. 1 is represented in time domain. Then in section *3.3*, IVA of the controllers is formulated as an optimization problem by finding the optimal initial states of the controller to minimize the performance index based on the tracking error. Design of performance index for short-seeking is discussed in section *3.4*. Simulation results are given in section *3.5*.

## 3.2 Closed- loop System in Time Domain

Assume that the dynamic model of the plant in discrete time is

$$\begin{aligned} x_v(k+1) &= A_v x_v(k) + B_v u(k) \\ y(k) &= C_v x_v(k) \end{aligned} \quad (2)$$

where $x_v \in \Re^{n_v}$ , $u$ and $y$ are the state vector, control input and output, respectively, $A_v \in \Re^{n_v^2}, B_v \in \Re^{n_v}$ and $C_v \in \Re^{1\times n_v}$ .

Suppose controller $C(z)$ can be expressed in time domain as follows:

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c e(k) \\ u(k) &= C_c x_c(k) + D_c e(k) \end{aligned} \quad (3)$$

where $x_c \in \Re^{n_c}$ , $A_c \in \Re^{n_c^2}, B_c \in \Re^{n_c}, C_c \in \Re^{1\times n_c}$ and $D_c \in \Re$

Combining the controller equation (3) and plant equation (2) and referring to Fig. 4, we can obtain the closed-loop system:

$$\begin{aligned} x(k+1) &= Ax(k) + B\,r \\ \mathrm{y}(k) &= Cx(k) \end{aligned} \quad (4)$$

where $$x\ (k) := \begin{bmatrix} x_v(k) \\ x_c(k) \end{bmatrix},$$

$$A = \left\{ \begin{bmatrix} A_v & B_v C_c \\ 0 & A_c \end{bmatrix} - \begin{bmatrix} B_v D_c \\ B_c \end{bmatrix} \begin{bmatrix} C_v & O \end{bmatrix} \right\},\ B = \begin{bmatrix} B_v D_c \\ B_c \end{bmatrix},\ C = \begin{bmatrix} C_v & O \end{bmatrix} \quad (5)$$

The resulting closed loop system (4) is stable because the track following controller has already been designed to stabilize the system. The error dynamics of the closed loop system is:

$$e_x(k+1) = Ae_x(k) \tag{6}$$

where $$e_x(k) = x(k) - x(\infty), \quad x(\infty) = (I-A)^{-1}Br \tag{7}$$

### 3.3 Initial Value Adjustment of the Controller

Now, the initial value adjustment of the controller can be formulated as the following optimization problem.

**Problem:** Consider the control system descried by Eq. (4). Find the initial value of the controller $x_c(0)$ that minimizes performance index $J$:

$$J = \sum_{k=0}^{\infty} e_x(k)^T Q\, e_x(k), \quad for\ Q > 0 \tag{8}$$

where $e_x(k)$ is as defined in (7).

**Theorem 1:** The initial value of the controller $x_c(0)$ that minimizes the performance index $J$ is:

$$x_c(0) = K_1 r + K_2\, x_v(0) \tag{9}$$

where $$K_1 = [P_{22}^{-1}P_{12}^T \quad I](I\text{-}A)^{-1}B, \quad K_2 = -P_{22}^{-1}P_{12}^T \tag{10}$$

and $P_{22}$ , $P_{12}$ are obtained by solving the Lyapunov Eq.

$$A^T PA - P = -Q \ , \quad P =: \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \tag{11}$$

**Proof:** Since $A$ is stable, (11) has a unique positive definite solution P>0, and $J$ can be transformed to:

$$J = e_x(0)^T P\, e_x(0)$$

where $$e_x(0) = x(0) - x(\infty) = \begin{bmatrix} x_v(0) \\ x_c(0) \end{bmatrix} - x(\infty) =: \begin{bmatrix} e_v(0) \\ e_c(0) \end{bmatrix} \tag{12}$$

So, $$J = e_v^T(0)\,P_{11}e_v(0) + 2e_v^T(0)\,P_{12}e_c(0) + e_c^T(0)\,P_{22}e_c(0)$$

It is easy to show that $\partial J/\partial x_c(0)=0$ is equivalent to $\partial J/\partial e_c(0)=0$. Solving $\partial J/\partial e_c(0)=0$, we get

$$e_c(0) = -P_{22}^{-1}P_{12}^T e_v(0) \tag{13}$$

From (12), (13) and (7), we conclude that J is minimized for $x_c(0)=K_1 r + K_2 x_k(0)$, where $K_1$, $K_2$ are as defined in (10). □

Noting that $x_v$ (0) may be assumed zero for 1-track seek, the initial value of the controllers turned out to be a simple form as:

$$x_c(0) = K_1\, r, \quad K_1 = [P_{22}^{-1}P_{12}^T \quad I](I\text{-}A)^{-1} B \tag{14}$$

### 3.4 Design of Performance Index Function

The performance index $J$ plays an important role in determining the initial value of the controller and thus the seeking performance. To obtain good performance in short seeking, the performance index is based on the squared summation of the error and smoothness of the control input for enhanced performance:

$$J = \sum_{k=0}^{\infty} [y(k) - y(\infty)]^2 + q[u(k) - u(\infty)]^2 \tag{15}$$

where $q \geq 0$ is a weighting factor.

**Lemma 1:** The performance index $J$ in (15) can be transformed into the standard quadratic form:

$$J = \sum_{k=0}^{\infty} e_x(k)^T\, Q\, e_x(k)$$

where $$Q = C^T C + qC_u{}^T C_u\,, \quad C_u = [-D_c C_v,\; C_c] \tag{16}$$

**Proof:** From Eq. (3),

$$u(\mathrm{k}) = C_c x_c(k) + D_c\, e(\mathrm{k}) = C_c x_c(k) + D_c(\, r \text{ - } \mathrm{y(k)}) \tag{17}$$

By substituting $y(\mathrm{k}) = C_v x_v(k)$ into (17), we get

$$u(k) = [-D_c C_v,\; C_c]x(k) + D_c r = C_u x(k) + D_c r$$

Thus, $$u(k) - u(\infty) = C_u[x(k) - x(\infty)] = C_u e_x(k)$$

Also, $$y(k) - y(\infty) = C[x(k) - x(\infty)] = C\, e_x(k)$$

Therefore, $$Q = C^T C + qC_u{}^T C_u$$ □

### 3.5 Simulations of Short- Seeking by IVA

Now we use the plant model (as in Fig. 2) and track-following controller (as in Eq. (1)), and assumed a density of 25.4 kTPI (one-track pitch is 1 *um*) for the simulation study. Simulation results of one-track seeking with IVA are shown in Fig.5 and Fig.6. In Fig. 5, the solid line shows the response of the proposed method and the dashed line shows the response of the output without initial value adjustment. From this figure, we can see that fast and smooth response in short seeking can be achieved by initial value adjustment of the controller.

Figure 6 indicates how the transient characteristics may be shaped by different weighting factors $q$ in the performance index. We can observe from Fig.6 that a smaller weighting factor $q$ (such as $q=0.8$), which allows for larger control inputs gives a fast response but may lead to overshoot of the head position, and a large weighting factor $q$ (such as $q=80$), which penalizes more the fluctuations of control input in the performance index, gives smoother but much slower response. As a result, we can tune the weighting factor *q* to achieve desired step-responses taking into account both speed and overshoot. Here we choose *q* to be *8*, where the overshoot is *0.02 track* and settling time is *1.0 ms*. Note that the *settling time* is defined as the time required for the read/write head to reach and stay within a steady error ε around the final target, where ε is 5% of the track pitch.

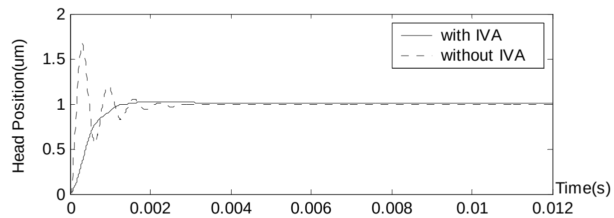



Fig.5 1-track seeking with/without initial value adjustment

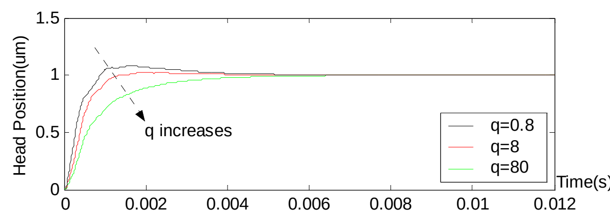



Fig. 6 1-track seeking with IVA for different weighting factor *q*.

From the simulations, we see that when seeking distance is very short, IVA can achieve desired transient by tunning the weighting factor and thus the initial value of the controller. However, as seeking distance becomes longer, there will be a compromise between the speed and overshoot in transient response, in other words, a fast rise will cause a large overshoot, and avoiding overshoot will makes response very slow.

For illustration, consider a seeking distance of 6 *track*. According to Eq. (14), the initial value of the controller is proportional to seeking distance *r*, so is the response of the head position. As shown in Fig.7, when the seeking distance becomes *r=6 track*, if we still choose *q=8*, then the overshoot scales up to *0.12 track* (*0.02 track*×*6*). The settling time due to this large overshoot becomes very long, which is *3.8 ms*. To avoid large overshoot, a much larger *q* (such as *q=80*) may be chosen, but it will make the response too slow. Thus, it is difficult in the present scheme to get a desired response with reduced settling time as seeking distance becomes longer. To overcome this problem, we propose a two-stage IVA scheme in the next section.

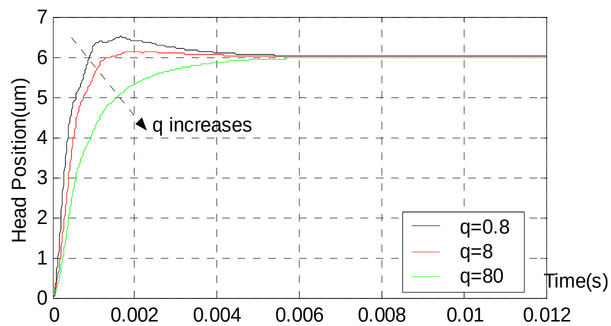



Fig.7 6-track seeking with IVA for different weighting factor *q*

## 4. SHORT- SEEKING WITH TWO-STAGE INITIAL VALUE ADJUSTMENT

The basic idea of two-stage IVA is that when the seeking distance becomes longer, seeking is divided into two stages, and IVA is applied in each of the two stages to obtain the desired transient in each stage. In the first stage, where the head is far away for the target track, IVA is used for a high-speed response; in the second stage, where the head is approaching the target track, IVA is used for a fast and smooth response without overshoot.

Note that almost all track following controllers include an integral action to improve the steady state error due to bias [9]. A typical track following controller is a proportional plus integral plus derivative (PID) controller, or a lead-lag compensator which consists of a lead filter and an integral action. Without loss of generality, the track following controller can be expressed as

$$C(z) = C_0(z)C_I(z) \tag{18}$$

where $C_I(z)$ is an integral action. In our example, the track following controller is

$$\begin{aligned} C(z) &= 4.0681\frac{(\mathrm{z-0.9057})(\mathrm{z-0.9756})(\mathrm{z-0.9905})}{(\mathrm{z-0.8664})(\mathrm{z-0.9577})(\mathrm{z-1})} \\ &= C_0(z)C_I(z) \end{aligned}$$

where

$$C_0(z) = 4.0681\frac{\mathrm{z-0.9057}}{\mathrm{z-0.8664}}\times\frac{\mathrm{z-0.9756}}{\mathrm{z-0.9577}}$$

$$C_I(z) = \frac{\mathrm{z-0.9905}}{\mathrm{z-1}} \tag{19}$$

In the two-stage IVA design, the integral action of the controller is inactivated in the first stage and activated only in the second stage as the head approaching the target to eliminate the steady-state error.

### 4.1 Overall Control Structure for Short Seeking by Two-stage IVA

Figures 8 and 9 show the schematic diagrams of short seeking control with two-stage IVA. The proposed method has the following features:

(1) The overall short seeking is divided into two stages, as referred to Fig. 9: in the first stage, where the head is far away from the target track, the integral action of

the controller, $C_I(z)$, is inactivated and controller $C_0(z)$ is used; in the second stage, where the head is approaching the target track, the integral action is switched on and the whole track following controller $C(z)$ is used.

(2) Reference input is set to be a step signal for the whole short-span seeking, i.e., $r(k)=r$ for $k \geq 0$, where $r$ is the seeking distance.

(3) Nonzero initial values are set to the corresponding controllers at the start instant of each stage to achieve desired short seek performance.

Some details of the two-stage design procedure are explained in the next subsections.

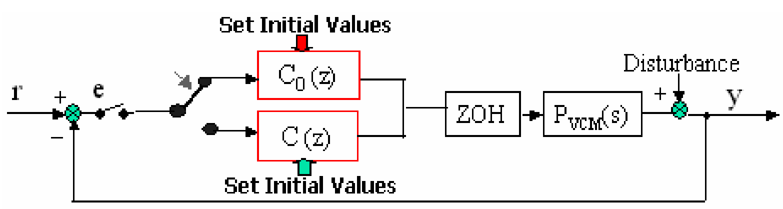
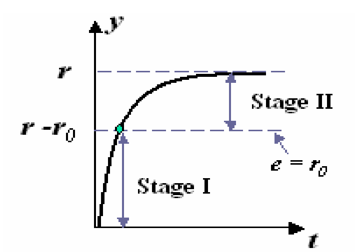
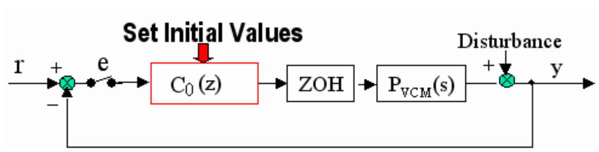


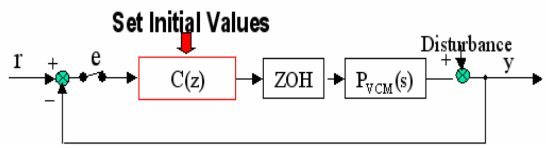

Fig.8 Short-seek control with Two-stage IVA




(a) Division of short seeking into two stages




(b) IVA in Stage *I*

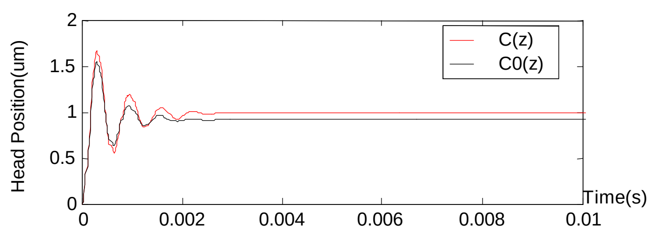



c) IVA in Stage *II*

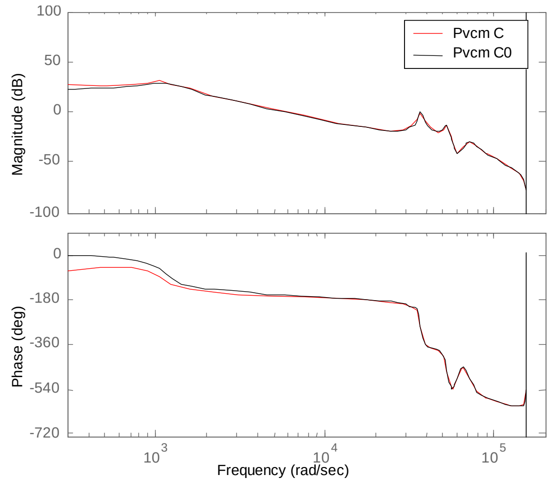
Fig.9 Short-seek with two-stage IVA

### 4.2 IVA in Stage I ( $e > r_0$ )

In the first stage, we use the servo system shown as Fig. 9 (b), which uses $C_0\,(z)$ as the controller. As is known, dropping the integral action $C_I\,(z)$ will lead to a nonzero steady state error, but will not appreciably affecting the stability of the closed loop system, which is demonstrated by Fig 10. In Fig.10 (a), the light (red) line is the time response of closed-loop system where the whole controller $C(z)$ is used. The dark (black) line is the response where $C_I\,(z)$ is dropped, and we observe a steady-state error as expected. In Fig.10 (b), the light (red) line is the frequency response of the open loop system $P_{VCM}\ C$, and the dark (black) line is the frequency response of $P_{VCM}\ C_0$, which confirms that stability of the closed-loop system is preserved when the integral action is dropped.




(a) Time response analysis




(b) Frequency responses analysis

Fig.10 Time response and frequency responses analysis with/without integral action

In *Stage I*, the initial value of $C_0$ can be obtained by following the same procedure as in Section *3* by first obtaining the closed loop system depicted in Fig.9 (b) in time domain as

$$\begin{aligned} x(k+1) &= Ax(k) + B\,r \\ y(k) &= Cx(k) + D\,r \end{aligned} \tag{20}$$

where $x(k){:=}[x_v(k)', x_{c0}(k)']'$, and $x_v$ and $x_{c0}$ are state vector of the plant and controller $C_0$, respectively.

Since this system is stable, the initial value of controller $C_0$ can be obtained using Theorem 1 as:

$$x_{c0}(0) = K_1 r$$

where $K_1{=}[P_{22}^{-1}P_{12}^{T}, I](I\text{-}A)^{-1}B$, and $P_{22}$, $P_{12}$ are obtained by solving the Lyapunov equation:

$$A^T PA - P = -Q\ ,\quad P =: \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix},\quad A \text{ is as defined in (20).}$$

The objective of IVA in *Stage I* is to get a rapid response. Thus, the weighting factor $q$ in the performance index should be set small (such as $q=0.8$).

### 4.3 IVA in Stage II ( $e \leq r_0$ )

The integral action is activated at the instance $e=r_0$, and in *Stage II*, we use the servo system in Fig. 9 (c), which is the same as the structure in Fig. 4, except that the initial value of the plants $x_v(0)$ is not zero here. Nonzero initial values are set to the controllers at the switching instance. The initial value here refers to the state at the switching instance. The initial values of $C(z)$ can be obtained by following the same procedure as in Section *3* by first obtaining the closed loop system in Fig.9 (c) in the form of (4), and then finding the optimal initial values of the controller using Theorem 1. Notice that the initial values of the controllers are determined by (9) instead of (14), due to the nonzero initial value of the plant.

The objective of IVA in *Stage II* is to get a fast and smooth response with little overshoot as the output approaches the target. Thus, the weighting factor $q$ in the performance index should be set larger (such as $q=20$).

### 4.4 Simulations of Short- Seeking by Two-stage IVA

The simulation result of *6-track* seeking using 2-stage IVA is shown in Fig.11. Here $r=6$ and we set $r_0$ to be 4. The light (red) line is the response in the first stage, where $q$ is chosen as $0.8$; the dark (black) line is the response in the second stage, where $q$ is chosen as

*20.* The settling time is greatly reduced to *1.4 ms* by two-stage IVA.

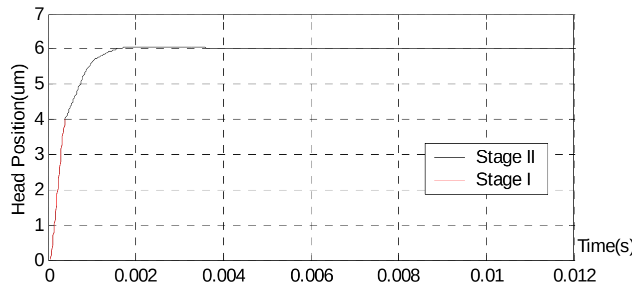



Fig. 11 6-*track* seeking by two-stage IVA

From above, we see that when the seeking distance becomes longer, two-stage IVA can achieve a desired response by inactivating the integral action in the first stage and assigning different values to the weighting factor *q* in different stages.

# 5. CONCLUSION

This report has discussed short seeking control methods based on track-following controllers and initial value adjustment for HDDs. IVA improves transient characteristics from the viewpoint of minimizing the performance index function. By incorporating the tracking error and the smoothness of control input in the performance index, IVA of the track-following controller can achieve desired transient in short seeking, when seeking distance is very short. To overcome the difficulty in compromising speed and overshoot as seeking distance becomes longer, a two-stage IVA scheme is designed to provide a high-speed movement in the first stage, and fast and precise positioning in the second stage.

Simulations for both 1-track seeking by IVA and short-span seeking (6-track) by two-stage IVA are performed. Simulation results confirmed that good short-seeking performances can be achieved using the proposed methods.

## ACKNOWLEDGEMENT

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